THE SIMULATION ANALYSIS OF MULTI-SHUTTLE AUTOMATED STORAGE AND RETRIEVAL SYSTEMS

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Abstract

Technological developments in warehouses have changed processes of storage operations, which reflect in short response times of the storage or retrieval of goods, the reduction of stocks and the volume of storage work as well as the automation of the entire warehouse management. Numerous companies are replacing costly and lasted traditional warehouses with automated storage and retrieval systems, which can be classified into unit-load and mini-load systems.

In this paper the simulation analysis of mini-load multi-shuttle systems is discussed and evaluated. Multi-shuttle systems are based on the quadruple and sextuple command cycle and could therefore achieve higher throughput capacities due to single-shuttle systems. Different analytical models are used by practitioners for designing multi-shuttle systems. The problem arises with the selection of the appropriate analytical model for which the condition of minimal differences with actual circumstances in practice is fulfilled. For the evaluation of the two well-known analytical models, the discrete event simulations have been used. Beside the evaluation of analytical models, the results of simulation analyses showed throughput improvements for triple-shuttle systems according to dual-shuttle systems.

The main objective of this paper is to determine the performance of presented models (analytical and simulation models) of multi-shuttle systems, which represents the main share and support in design process of multi-shuttle automated storage and retrieval systems.

Keywords: Mini-load AS/RS, Multi-shuttle systems, Discrete event simulation, Performance analysis.

Presenting Author's biography

Tone Lerher is an assistant professor, holding a Ph.D. degree, employed at the Faculty of Mechanical Engineering, University of Maribor. His field of research refers to the pedagogical topic of "Technology in the logistics" and "Devices, systems and constructions for transport". As a senior researcher he has taken part in applicable and fundamental projects in his career. He is also a member of the professional association European Association for Traffic, Transport and Business Logistics. Last but not least, he is the author and co-author of original scientific, scientific and technical papers in his field of research.

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1 Introduction

The successful usage of a warehouse depends upon the appropriate design, selection and operation of the type of warehouse and material handling equipment. When designing warehouses, the warehouse planner has to strike a balance between flexibility, layout configuration, storage density and throughput capacity in order to achieve an effective design at a minimum cost. Estimates indicate that, depending on the type of industry, at least 25 % of the cost of a product is represented by the physical movement. Therefore every decision related to warehousing can reduce the logistics cost. An important part of warehouses is presented by mini-load Automated Storage and Retrieval Systems (AS/RS), which are widely used in many fields of automotive, chemical, pharmaceutics industry, where the basic Transport Unit Load (TUL) is presented by a storage container. The mini-load AS/RS is composed of multiple parallel aisles of Storage Racks (SR), Storage and Retrieval machine (S/R machine) intended for each aisle, Input and Output (I/O) location and accumulating conveyors. Advantages of the application of AS/RS are: efficient utilization of the warehouse space, reduction of damage and loss of goods, increased control upon storage and retrieval of goods and decrease in the number of warehouse workers. On the other hand the mini-load AS/RS require a high initial investment and they are rather inflexible to meet future demands. Therefore a careful design of mini-load AS/RS is crucial for the AS/RS to be successful. The performance of the mini-load AS/RS is often evaluated by the number of TUL per hour which may be stored and the number of TUL which may be retrieved – the throughput capacity of the system. Due to the increasing requests for higher throughput capacities and shorter response times in handling the orders, special designs of S/R machines which can carry several TUL simultaneously have been constructed. Many warehousing equipment producers have begun to offer such multi-shuttle S/R machines that can receive up to three TUL simultaneously and consequently higher throughput capacities can be achieved. AS/RS have been the subject of many researchers over the past few years. Their intensive development has begun with the development of the informational and computer science, which represents an important part of the warehouse operation. Hausman et al. [1] have analyzed AS/RS only for square-in-time racks (SIT racks). They have analyzed the Single Command Cycle (SC) and different storage strategies, e.g. random storage, throughput-based storage and class-based storage. Graves et al. [2] have developed methods for determination of the Dual Command Cycle (DC) with different storage strategies for SIT racks. The impacts of rack geometry for non-SIT rack on travel times have been analyzed by Bozer and White [3]. They have developed analytical models for calculating SC and DC. Their model is based on randomized storage and retrieval with different Input Output (I/O) configurations of the input queue. Han at al. [4] have shown that the throughput capacity can be increased by replacing the "First Come, First Served" (FCFS) retrieval sequencing with a new "Nearest Neighbor" (NN) heuristics policy. According to their observations, a 50 % or more decrease in the Travel Between time component (TB) of DC leads to an increase in throughput of 10 – 15 %. Such an increase in throughput could help to handle peak demand in the operation phase or even to eliminate an aisle, which leads to considerable savings.

With regard to the literature survey and current work, the majority of researchers have analyzed single-shuttle AS/RS. Throughput capacities of the single-shuttle AS/RS are limited with maximal technical characteristics of SR machines and optimal Storage Racks (SR) geometry. Hence SR machines that can storage and retrieve several TUL simultaneously (multi-shuttle systems) have to be used to increase the throughput capacity. Multi-shuttle AS/RS are divided into dual-shuttle and triple-shuttle AS/RS. In dual-shuttle AS/RS, the S/R machine can perform up to two storages and two retrievals in a cycle, which is called Quadruple command Cycle (QC). Further on, in triple-shuttle AS/RS the S/R machine can perform up to three storages and three retrievals in a cycle, which is called Sextuple command Cycle (STC). The main problem with the multi-shuttle AS/RS is how to find out an appropriate heuristics that the condition of minimal travel times will be fulfilled. In addition to our research, Keserla and Peters [5] have presented an analysis of dual shuttle AS/RS. They have presented the heuristics for minimizing the Travel Between time (TB) component for DC and shown that the throughput improvement using QC due to DC is in the range of 40 – 45 %. Analytical models under multi-shuttle AS/RS have also been presented by Meller and Mungwatana [6]. Within the storage operation of QC and STC, they have used general and modified QC and STC with NN request selection rule. Their analytical models are based on the assumption that the S/R machine all the time travels with constant velocity (the basis of their work is analytical models of Bozer & White [3]). On the other hand Gudehus [7] has presented an analytical model for multi-shuttle AS/RS in which the storage location assignment policy and the request selection rule are based on *Strategy x* heuristics. His analytical models are based on the assumption that the S/R machine travels with variable velocity.

In this paper, the mini-load AS/RS for the multi-shuttle systems are presented and evaluated. Since the existing analytical models for the multi-shuttle systems apply to the assumption of uniform velocity [6] and have several simplifications and constraints [7], the discrete event simulations is used to evaluate the real performance and the efficiency of the multi-shuttle systems. The main objective of our research is to determine the real efficiency of the multi-shuttle systems (simulation model) in comparison with analytical models of Meller and Mungwatana [6] and Gudehus [7]. The results of our research therefore represent the support in designing processes of mini-load AS/RS.

2 Analytical travel time models of multi-shuttle systems

2.1 Assumptions and notations

The analytical travel time models of multi-shuttle AS/RS are based on the following assumptions and notations:

- The storage rack is considered to be a continuous rectangular pick face, where the I/O point is located at the lower left-hand corner of the storage rack.
- The storage rack Length (L) and Height (H) as well as the S/R machine velocity in the horizontal v_x and vertical v_z directions are known.
- The S/R machine travels simultaneously in the horizontal x and in the vertical direction y. For calculating the travel time, constant (Meller and Mungwattana [6]) and variable (Gudehus [7]) velocities were used for the horizontal and vertical travel.
- Pickup and deposit times associated with handling the TUL are assumed to be constant and therefore they could be easily added to the cycle time expressions.
- The *S/R* machine operates either on a Quadruple Command (QC) or Sextuple Command (STC) cycle.
- Nearest Neighbor "NN" [6] and the *Strategy x* [7] storage policy were implemented in the multi-shuttle system.

Symbols:

v_x – the S/R machine velocity in the horizontal direction,

v_y – the S/R machine velocity in the vertical direction,

a_x – the S/R machine acceleration and deceleration in the horizontal direction,

a_y – the S/R machine acceleration and deceleration in the vertical direction,

H – the height of the storage rack,

L – the length of the storage rack,

SF – the scaling factor,

b – the shape factor,

NN – nearest neighbor policy,

QC – quadruple command cycle,

STC – sextuple command cycle,

$E(SW_{\mathrm{I/O}}^{s1})$ – the expected travel time from the I/O station to s_1,

$E(TB_{s1}^{s2})$ – the expected travel time between s_1 and s_2,

$E(TB_{s2}^{s3})$ – the expected travel time between s_2 and s_3,

$E(TB_{s3}^{r1})$ – the expected travel time between s_3 and r_1,

$E(TB_{r1}^{r2})$ – the expected travel time between r_1 and r_2,

$E(TB_{r2}^{r3})$ – the expected travel time between r_2 and r_3,

$E(SA_{r3}^{\mathrm{I/O}})$ – the expected travel time from r_3 to the I/O station,

$E(QC^{NN})$ – the expected total travel time of QC with the NN policy,

$E(STC^{NN})$ – the expected total travel time of STC with the NN policy,

α, β – rack factors,

Θ – discrete function,

η – the efficiency of the S/R machine,

I_n^1 – the expected travel time from the I/O station to the first of n-open locations P(1),

B_n^1 – the expected acceleration/deceleration time from the I/O station to the first of n-open locations P(1),

I_n^i – the expected travel time between two successive P(i) and P(i+1) locations,

B_n^i – the expected acceleration/deceleration time between two successive P(i) and P(i+1) locations.

$E(QC^{Strat.x})$ – the expected total travel time of QC with the Strategy x policy,

$E(STC^{Strat.x})$ – the expected total travel time of STC with the Strategy x policy,

2.2 Expected travel time in the multi-shuttle systems based on constant velocity

The model for the calculation of the expected travel time in the twin- and triple-shuttle systems under the NN storage policy is presented in the continuation [6].

- **Estimation of travel time with regard to STC and the NN storage policy.**

The expected travel time of STC with the NN storage policy is modeled as follows:

The time to reach the end t_x and the top t_y of the SR:

$$t_x = \frac{L}{v_x}; t_y = \frac{H}{v_y} \tag{1}$$

The scaling factor:

$$SF = \max\left(t_x, t_y\right) \tag{2}$$

The shape factor (dimensionless):

$$b = \min\left(\frac{t_x}{T}, \frac{t_y}{T}\right) \tag{3}$$

Step 1 – within three open locations (s_1, s_2 and s_3) the S/R machine moves from the I/O station to the closest location, say s_1. The expected travel time from the I/O station to s_1 is equal to the expected shortest one-way travel time with 3 open locations (m = 3).

$$E\left(SW_{\mathrm{I/O}}^{s1}\right) = \left(\frac{1}{4} + \frac{b^2}{2} - \frac{2b^3}{5} + \frac{3b^4}{28}\right) \tag{4}$$

Step 2 – after storing the first unit load, the S/R machine moves to the second storage location s_2, since it is closer to s_1 than s_3. The expected travel time between s_1 and s_2 is estimated to be the expected shortest travel-between time from [4] with m = 2.

$$E\left(TB_{s1}^{s2}\right) = \left(\frac{b^2}{3} - \frac{31b^3}{105} + \frac{5b^4}{42} - \frac{11b^5}{630} + \frac{1}{5}\right) \tag{5}$$

Step 3 – after storing the second unit load, the S/R machine moves to the third storage location s_3. The expected travel time between s_2 and s_3 is estimated as the single-shuttle expected travel-between time from [3].

$$E\left(TB_{s2}^{s3}\right) = \left(\frac{1}{3} + \frac{b^2}{6} - \frac{b^3}{30}\right) \tag{6}$$

Step 4 – from s_3 the S/R machine moves to the closest of the three retrieval points (r_1, r_2 and r_3), say, r_1. The expected travel time between s_3 and r_1 is expressed by evaluating the following expression [4] with m = 3.

$$E\left(TB_{s3}^{r1}\right) = \int_0^1 mz\left(1 - F\left(z\right)^{m-1}\right) \cdot f\left(z\right) dz \tag{7}$$

Step 5 – from the first retrieval location r_1 the S/R machine moves to the closest of the last two retrieval locations, say r_2. The expected travel time between r_1 and r_2 is estimated as the expected travel time between s_1 and s_2, which is the minimum of two expected travel-between times.

$$E\left(TB_{r1}^{r2}\right) = \left(\frac{b^2}{3} - \frac{31b^3}{105} + \frac{5b^4}{42} - \frac{11b^5}{630} + \frac{1}{5}\right) \tag{8}$$

Step 6 – the S/R machine then moves to the third retrieval point r_3. The expected travel time between r_2 and r_3 is estimated to equal the expected travel time between s_2 and s_3.

$$E\left(TB_{r2}^{r3}\right) = \left(\frac{1}{3} + \frac{b^2}{6} - \frac{b^3}{30}\right) \tag{9}$$

Step 7 – Finally, the S/R machine returns to the I/O station. The expected travel time from r_3 to the I/O station is estimated to equal the expected one-way travel time.

$$E\left(SA_{r3}^{\mathrm{I/O}}\right) = \left(\frac{1}{2} + \frac{b^2}{6}\right) \tag{10}$$

The expected total travel time of STC with the NN storage policy is estimated as:

$$\begin{aligned} E\left(STC^{NN}\right) = {} & E\left(SW_{\mathrm{I/O}}^{s1}\right) + E\left(TB_{s1}^{s2}\right) + E\left(TB_{s2}^{s3}\right) + \\ & E\left(TB_{s3}^{r1}\right) + E\left(TB_{r1}^{r2}\right) + E\left(TB_{r2}^{r3}\right) + \\ & E\left(SA_{r3}^{\mathrm{I/O}}\right) \end{aligned} \tag{11}$$

The above analytical model is valid for three open locations (m = 3); however it can be generalized to n open locations by replacing the current value of m.

Therefore the expected total travel time of QC with the NN storage policy is estimated as:

$$\begin{aligned} E\left(QC^{NN}\right) = {} & E\left(SW_{\mathrm{I/O}}^{s1}\right) + E\left(TB_{s1}^{s2}\right) + E\left(TB_{s2}^{r1}\right) + \\ & E\left(TB_{r1}^{r2}\right) + E\left(SA_{r2}^{\mathrm{I/O}}\right) \end{aligned} \tag{12}$$

For a detailed explanation and insight into the analytical model see the paper by Meller and Mungwatana [6].

2.3 Expected travel time in the multi-shuttle systems based on variable velocity

In the continuation the model for the calculation of the expected travel time in twin- and triple-shuttle systems with regard to *Strategy x* storage policy (the *Strategy x* policy is similar to the NN policy) is presented [7].

- **Estimation of travel time under STC and the *Strategy x* storage policy.**

The expected travel time of STC with the *Strategy x* storage policy is modeled as follows:

The SR factors (dimensionless):

$$\begin{aligned} & \alpha = \frac{H \cdot v_x}{L \cdot v_y} \quad \Theta = 1 \text{ for } \alpha < 1 \text{ and } \Theta = 0 \text{ for } \alpha > 1 \\ & \beta = \frac{v_y}{a_y}\left(\frac{v_x}{a_x} - \frac{v_y}{a_y}\right) \end{aligned} \tag{13}$$

Step 1 – the expected travel time from the I/O station to the first of n-open locations P(1).

$$I_n^1 = \frac{H}{2v_y}\left[1 + \frac{2}{(n+1)(n+2)} \cdot \frac{1}{\alpha^2}\left(1 - \Theta\left(1 - \alpha\right)^{n+2}\right)\right] \tag{14}$$

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Step 2 – the expected acceleration/deceleration time from the I/O station to the first of *n*-open locations P(1).

$$B_n^1 = \frac{v_y}{a_y}\left[1 + \frac{\beta}{(n+1)}\cdot\frac{1}{\alpha}\left(1-\Theta\left(1-\alpha\right)^{n+1}\right)\right] \quad (15)$$

Step 3 – the expected travel time between two successive P(*i*) and P(*i*+1) locations,

$$I_n^i = \frac{H}{3v_y}\left[\begin{matrix}1 + \frac{6}{(n+1)(n+2)}\cdot\frac{1}{\alpha^3}\cdot \\ \left(\alpha - \frac{1}{n+3}\left(1-\Theta\left(1-\alpha\right)^{n+3}\right)\right)\end{matrix}\right] \quad (16)$$

Step 4 – the expected acceleration/deceleration time between two successive P(*i*) and P(*i*+1) locations.

$$B_n^i = \frac{v_y}{a_y}\left[\begin{matrix}1 + \frac{2\beta}{(n+1)}\cdot\frac{1}{\alpha^2}\cdot \\ \left(\alpha - \frac{1}{n+2}\left(1-\Theta\left(1-\alpha\right)^{n+1}\right)\right)\end{matrix}\right] \quad (17)$$

The expected total travel time of STC with the *Strategy x* storage policy is estimated as:

$$E\left(STC^{Strat.\,x}\right) = 2\left(I_n^1 + B_n^1\right) + 5\left(I_n^i + B_n^i\right) \quad (18)$$

The above analytical model is valid for three open locations; however it can be generalized to *n* open locations.

The expected total travel time of QC with the *Strategy x* storage policy is estimated as:

$$E\left(QC^{Strat.\,x}\right) = 2\left(I_n^1 + B_n^1\right) + 3\left(I_n^i + B_n^i\right) \quad (19)$$

3 Simulation model of multi-shuttle AS/RS

To facilitate the performance evaluation and comparison of the multi-shuttle AS/RS, the discrete event simulation was used.

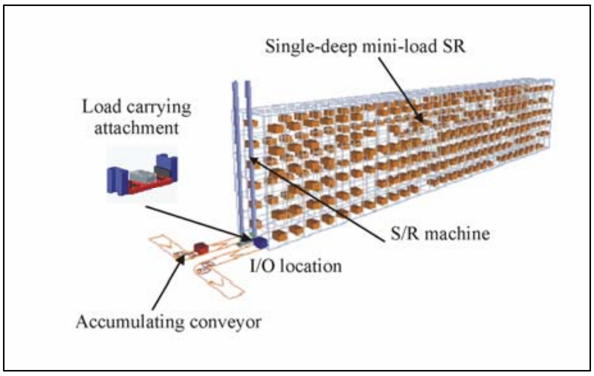

Fig. 1 The simulation model of multi-shuttle AS/RS

The simulation model of multi-shuttle AS/RS consists of two lines (single-deep) of SR, SR machine, I/O location and other manipulation equipment [8].

3.1 The definition of storage strategies in the simulation model

The algorithm for performing quadruple and sextuple command cycles in the simulation model of multi-shuttle system is written in the following way [8], [9], [10]:

s_i – the i^{th} open location for storing the TUL,

r_j – the j^{th} retrieval assignment of the TUL,

S_i – the set of i^{th} open storage locations in the storage list from SR_i ($i = 1, \ldots, m$),

R_j – the set of j^{th} closed storage locations in the retrieval list from SR_j ($j = 1, \ldots, n$).

- **Quadruple command cycle**

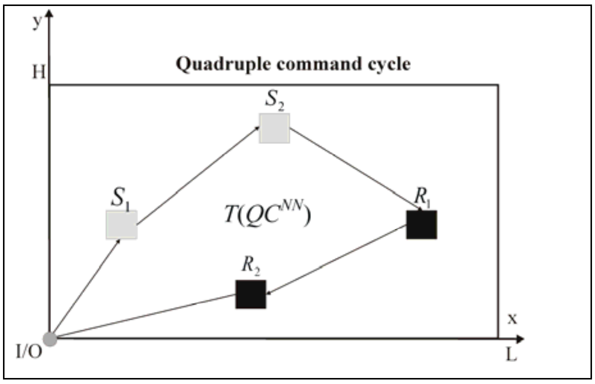

Fig. 2 The quadruple command cycle

$s_i \in \left(S_i; i = 1,\ldots,m\right)$ and $r_j \in \left(R_j; j = 1,\ldots,n\right)$

1) If $S_i \geq 1$, then select $s_1 \in S_i$ on the NN basis:

Perform the storage assignment in s_1:

$$\begin{matrix} S_i \leftarrow S_i - \{s_1\} \\ R_j \leftarrow R_j + \{s_1\} \end{matrix} \quad (20)$$

2) If $S_i \geq 1$, then select $s_2 \in S_i$ on the NN basis:

Perform the storage assignment in s_2:

$$\begin{matrix} S_i \leftarrow S_i - \{s_2\} \\ R_j \leftarrow R_j + \{s_2\} \end{matrix} \quad (21)$$

3) If $R_i \geq 1$, then select $r_1 \in R_j$ on the NN basis:

Perform the retrieval assignment in r_1:

$$\begin{matrix} R_j \leftarrow R_j - \{r_1\} \\ S_i \leftarrow S_i + \{r_1\} \end{matrix} \quad (22)$$

4) If $R_i \geq 1$, then select $r_2 \in R_j$ on the NN basis:

Perform the retrieval assignment in r_2:

$$\begin{aligned} R_j &\leftarrow R_j - \{r_2\} \\ S_i &\leftarrow S_i + \{r_2\} \end{aligned} \tag{23}$$

The mean quadruple command travel time T(QC) consists of the mean single command travel time T(SC) and three mean travel between time components T(TB).

$$T(QC^{NN}) = T(SC) + 3 \cdot T(TB) \tag{24}$$

- **Sextuple command cycle**

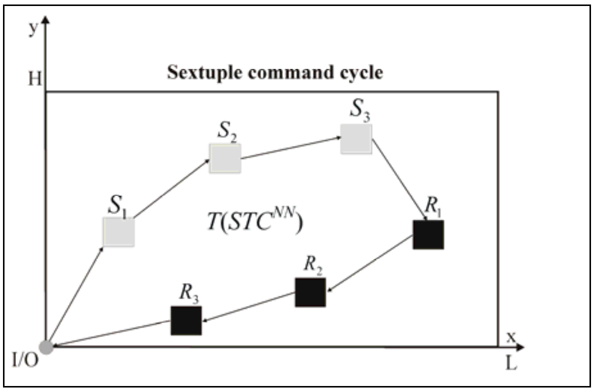

Fig. 3 The sextuple command cycle

$s_i \in (S_i; i = 1,...,m)$ and $r_j \in (R_j; j = 1,...,n)$

1) If $S_i \geq 1$, then select $s_1 \in S_i$ on the NN basis:

Perform the storage assignment in s_1 :

$$\begin{aligned} S_i &\leftarrow S_i - \{s_1\} \\ R_j &\leftarrow R_j + \{s_1\} \end{aligned} \tag{25}$$

2) If $S_i \geq 1$, then select $s_2 \in S_i$ on the NN basis:

Perform the storage assignment in s_2 :

$$\begin{aligned} S_i &\leftarrow S_i - \{s_2\} \\ R_j &\leftarrow R_j + \{s_2\} \end{aligned} \tag{26}$$

3) If $S_i \geq 1$, then select $s_3 \in S_i$ on the NN basis:

Perform the storage assignment in s_3 :

$$\begin{aligned} S_i &\leftarrow S_i - \{s_3\} \\ R_j &\leftarrow R_j + \{s_3\} \end{aligned} \tag{27}$$

4) If $R_i \geq 1$, then select $r_1 \in R_j$ on the NN basis:

Perform the retrieval assignment in r_1 :

$$\begin{aligned} R_j &\leftarrow R_j - \{r_1\} \\ S_i &\leftarrow S_i + \{r_1\} \end{aligned} \tag{28}$$

5) If $R_i \geq 1$, then select $r_2 \in R_j$ on the NN basis:

Perform the retrieval assignment in r_2 :

$$\begin{aligned} R_j &\leftarrow R_j - \{r_2\} \\ S_i &\leftarrow S_i + \{r_2\} \end{aligned} \tag{29}$$

6) If $R_i \geq 1$, then select $r_3 \in R_j$ on the NN basis:

Perform the retrieval assignment in r_3 :

$$\begin{aligned} R_j &\leftarrow R_j - \{r_3\} \\ S_i &\leftarrow S_i + \{r_3\} \end{aligned} \tag{30}$$

The mean sextuple command travel time T(STC) consists of the mean single command travel time T(SC) and five mean travel between time components T(TB).

$$T(STC^{NN}) = T(SC) + 5 \cdot T(TB) \tag{31}$$

3.2 The simulation model

The simulation model of multi-shuttle AS/RS begins with the process that indicates all storage locations in the storage rack. After creating the list of free storage locations, the TUL enter the simulation model 2 (3) with the arrival frequency λ. The arrival process is illustrated with the accumulating conveyor, on which TUL are arriving and departing the system (see Fig. 1). After the concluded transport with accumulating conveyor, TUL are situated in the I/O location, which lies at the lower left hand corner of the SR. Next the TUL receive a sign, which is dedicated to the storage location s_i in the SR. The SR machine picks up TUL from the I/O location, loads them into the shuttles, and moves to the prescribed storage locations in the SR. For the storage operation the NN storage assignment policy was used (see chapter 3.1). TUL that have been stored are then inscribed by a computer on a waiting list. The storage operation of TUL goes on until the warehouse reaches a certain degree of fillgrade (e.g. 85 %). In case of overriding the degree of fillgrade, a new retrieval process of TUL begins. For the retrieval operation the NN request selection rule was used (see chapter 3.1). Next the S/R machine travels to the retrieval location of the TUL and delivers them to the I/O location. For every single type of AS/RS, a special simulation model has been designed in such a way that the general simulation model has been supplemented.

3.3 The multi-shuttle AS/RS under study

According to the references and practical experiences it has been established that different layouts of the SR, the efficiency of the S/R machine and control policies have a tremendous influence on the average travel time and consequently on the performance – throughput capacity. Therefore, six different layouts of the SR have been used in our analyses [11]. According to the efficiency of the S/R machine, *Stöcklin* automated S/R machine has been used for performing QC and STC cycles (v_x = 6 m/s, v_y = 2 m/s, a_x = 3 m/s^2, a_y = 2 m/s^2, η = 0,95) [12]. It must be emphasized that the simulation model is based on traveling with variable velocity. Two types of velocity profiles can be distinguished depending on whether the obtained peak velocity is less than v_{max} (type I.) or equal to v_{max} (type II.). It can be verified that $T < 2v_{max}/a$ for type I. and $T > 2v_{max}/a$ for type II.

- S/R machine traveling for type I. ($T < 2v_{\max}/a$)

$$s = \int_0^T v(t)dt$$
$$s = \int_0^{t_p} a \cdot t dt + \int_{t_p}^{T} -a(t-T)dt = \frac{a \cdot T^2}{4} \quad (32)$$

- S/R machine traveling for type II ($T > 2v_{\max}/a$)

$$s = \int_0^T v(t)dt$$
$$s = \int_0^{t_p} a \cdot t dt + \int_{t_p}^{T-t_p} v dt + \int_{T-t_p}^{T} -a(t-T)dt \quad (33)$$
$$s = v_{\max} \cdot T - \frac{v_{\max}^2}{a}$$

Like the layout of the SR and the efficiency of the S/R machine, the control policy also has a significant influence on the average travel time. For the multi-shuttle AS/RS, the NN storage policy and NN retrieval assignment policy were applied (see chapter 3.1).

Tab. 1. Dimensions of six different types of SR [11]

SR	N_x	N_y	L (m)	H (m)
SR I.	60	19	30	6,08
SR II.	40	41	20	13,12
SR III.	90	32	45	10,24
SR IV.	120	41	60	13,12
SR V.	140	22	70	7,04
SR VI.	160	47	80	15,04

4 Analysis of results and discussion

The mean quadruple T(QC) and sextuple T(STC) command travel times (sec.) and the performance *Pf* of the multi-shuttle AS/RS, which are presented in the following Table 1, are stated on the basis of the performed analyses. Analyses have been conducted for the presented analytical models of Meller and Mungwatana [6], Gudehus [7] and the simulation model of multi-aisle AS/RS [13]. Six different layouts of the SR according to the chosen velocity profile of the S/R machine with regard to the presented NN storage policy have been used. In order to receive the best representative average for the average travel time, the simulation results presented in Table 1 correspond to 1.000.000 cycles for every type of SR. The performance of the multi-shuttle AS/RS presents the number of transactions (stores and retrievals) that the S/R machine can perform in a given time period. The *Pf* is inversely dependent on the mean travel time. Times which originate within manipulation of the TUL (identification of TUL, pickup and deposit times for TUL) must also be considered by determination of the performance.

$$T(QC)_{cycle} = T(QC) + \sum_1^m T_m \quad T_m = 22 \text{ sec.}$$
$$T(STC)_{cycle} = T(STC) + \sum_1^m T_m \quad T_m = 30 \text{ sec.} \quad (34)$$

In the QC and STC, the storage and retrieval operations are performed for a total of four and six transactions. Therefore the *Pf* in the selected time unit T (e.g. 1 hour) for the QC and STC is given as follows:

$$Pf_{QC} = \left(\left(\frac{T}{T(QC)_{cycle}} \right) \cdot 4 \right) \cdot \eta \quad \left[\frac{TUL}{h} \right]$$
$$Pf_{STC} = \left(\left(\frac{T}{T(STC)_{cycle}} \right) \cdot 6 \right) \cdot \eta \quad \left[\frac{TUL}{h} \right] \quad (35)$$

4.1 The comparison of the mean travel time and the performance of multi-shuttle AS/RS

(*i*) Analytical model of Meller in Mungwatana [6]

The mean QC and STC travel times and consequently the performance of the multi-shuttle system show the expressive deviation from the results of the simulation analysis. The expressive deviations of the mean travel time are a consequence of the assumption that the S/R machine travels all the time with a constant velocity. The analytical model is therefore applicable when we have long and high SR (large *L* and large *H*) and when the influences of acceleration and deceleration are neglected due to travelling of the S/R machine with a constant velocity. In Figures 4 and 5, the deviation of the mean travel time is in the range of 50 %, which consequently has the effect that the performance of the multi-shuttle AS/RS is over-evaluated for 35 %.

(*ii*) Analytical model of Gudehus [7]

This analytical model is based on the variable velocity and uses the assumption that the S/R machine travels all the time with the velocity-time dependence II. (see chapter 3.3). The deviation of the mean travel time is a consequence of the individual simplification and constraints in the model. In Figures 4 and 5, the deviation of the mean travel time is in the range of 20 %, which consequently has the effect that the performance is over-evaluated for 10 %.

Tab. 1. The performance of multi-shuttle AS/RS using different models

SR *i*	Simulations				Gudehus [7]				Meller at al. [6]			
	QC		STC		QC		STC		QC		STC	
	T_{QC} (sec.)	Pf_{QC} (1/h)	T_{STC} (sec.)	Pf_{STC} (1/h)	T_{QC} (sec.)	Pf_{QC} (1/h)	T_{STC} (sec.)	Pf_{STC} (1/h)	T_{QC} (sec.)	Pf_{QC} (1/h)	T_{STC} (sec.)	Pf_{STC} (1/h)
I.	19,9 (0,00)	326 (0,00)	27,1 (0,00)	359 (0,00)	16,2 (-18,6)	358 (9,8)	22,5 (-17,0)	391 (8,9)	8,7 (-56,3)	446 (36,8)	12,3 (-54,6)	485 (35,1)
II.	20,9 (0,00)	319 (0,00)	28,4 (0,00)	351 (0,00)	19,7 (-5,8)	328 (2,8)	26,9 (-5,3)	361 (2,9)	10,8 (-48,3)	417 (30,7)	15,3 (-46,1)	453 (29,0)
III.	26 (0,00)	285 (0,00)	35,3 (0,00)	314 (0,00)	21,1 (-18,9)	317 (11,2)	29,3 (-17,0)	346 (10,2)	13,6 (-47,7)	384 (34,7)	19,1 (-45,9)	418 (33,1)
IV.	31,7 (0,00)	255 (0,00)	42,9 (0,00)	281 (0,00)	25,4 (-19,9)	289 (13,3)	35,1 (-18,2)	315 (12,1)	17,8 (-43,9)	344 (34,9)	25,2 (-41,3)	372 (32,4)
V.	33,3 (0,00)	247 (0,00)	45 (0,00)	274 (0,00)	24,7 (-25,8)	293 (18,6)	34,5 (-23,3)	318 (16,1)	17,1 (-48,7)	350 (41,7)	24,7 (-45,1)	375 (36,9)
VI.	38,5 (0,00)	226 (0,00)	52,2 (0,00)	250 (0,00)	30 (-22,0)	263 (16,4)	41,5 (-20,5)	287 (14,8)	22,6 (-41,3)	307 (35,8)	32 (-38,7)	331 (32,4)

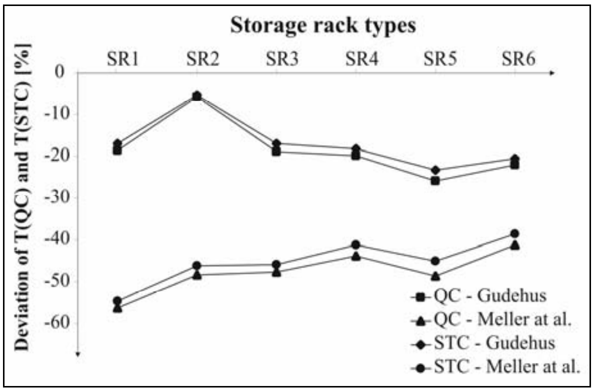

Fig. 4 The comparison of the average quadruple and sextuple travel time

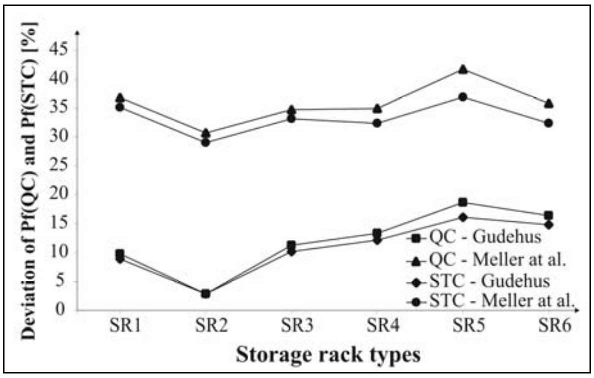

Fig. 5 The performance comparison of multi-shuttle AS/RS

(*iii*) Simulation model of multi-shuttle AS/RS

The simulation model of multi-shuttle AS/RS is based on the velocity time dependence I. and II. (see chapter 3.3) and the NN assignment policy. In the simulation model, the S/R machine “sway” and the “creeping” time were not considered.

According to the distribution of the mean quadruple and sextuple travel times, it can be estimated that the SR geometry has a tremendous impact on the travel time. It is obvious that the lowest mean travel times belong to SR with small *L* and small *H*, while the largest mean travel times belong to the type of SR with large *L* and large *H*. The comparison of QC and STC indicates the increase in the mean travel times. This can be explained with the fact that the SR machine requires more time to visit all storage locations under STC, due to QC. It must be emphasized that the mean travel time would be even higher if the NN storage policy was not used for the storage and retrieval request in the multi-shuttle AS/RS. The conclusion is that the SR geometry, velocity profile of the SR machine and the selection of dual or triple-shuttle system are the most significant parameters and have a major influence on the mean travel times. According to the performance of multi-shuttle AS/RS, the STC in comparison with QC enables higher throughput capacities. The throughput improvement of the triple-shuttle system is evident in comparison with the dual-shuttle system and even more so with the single-shuttle system. For example, in comparison with the single-shuttle system the improvement in throughput could eliminate one or more aisles and therefore large savings can be achieved. Even though the multi-shuttle AS/RS are more expensive than the single-shuttle AS/RS, this increase may be lower than the increase in savings due to the elimination of aisles. Similar conclusions have also been presented by authors Keserla and Peters [5]. Therefore our results could help the warehousing planners to decide in the early stage of the project which type of multi-shuttle AS/RS will be installed with regard to the TUL turnover.

5 Conclusions

In this paper, the performance analysis applying analytical models and the simulation model of the multi-shuttle AS/RS is presented. Because of the simplification and constraints of the presented analytical models [6], [7] real operating characteristics of the S/R machine have been used in the simulation model. The proposed models are based on the sequencing storage and retrieval requests on the nearest neighbor and Strategy x assignment policies. Various elements of the multi-shuttle AS/RS have been examined, such as the layout of the SR and the efficiency of the S/R machine in order to investigate the efficiency of the presented analytical models in comparison with the simulation model of multi-shuttle AS/RS.

According to the analytical models of Meller and Mungwatana [6], the largest deviation of the mean travel time occurs in the range from 38 % to 56 %, depending of the layout of the SR. In the case of analytical model of Gudehus [7], the mean travel time occurs in the range from 5 % to 25 %. Therefore the models by Meller and Mungwatana [6] and Gudehus [7] are over-evaluated (35 % [6], 10 % [7]), which indicates a difficulty in planning the multi-shuttle AS/RS in practice. Thus when designing multi-shuttle AS/RS, the simulation model will demonstrate the best performances with regard to real conditions and could therefore be a very useful tool for professionals in practice.

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