THE PSEUDO-OUTPUT ERROR IDENTIFICATION ALGORITHM: EFFECT OF THE STATIONARY FILTER

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Abstract

A new off-line optimization approach for system identification, known as the Pseudo-Output Error (POE) algorithm, is based on the introduction of a stationary filter in order to compute the sensitivity functions. The selection of the filter is crucial so that the POE algorithm converges to the desired values of the real system. Generally, it is possible to have a priori knowledge of the desired system such as input-output measurements, harmonic studies or the step input response. The selected stationary filter is a low pass filter. Its order is assumed to be the same as the order of the model and the corresponding pass band is a priori information. In this context, the effect of the bandwidth, defined in terms of the cut-off frequency of the stationary filter, on the convergence of the POE algorithm is studied and is analyzed. The results of the performed Monte-Carlo simulations show that the convergence of the POE algorithm to the desired values of the system to be identified is affected by the selection of the cut-off frequency of the stationary filter. It is more appropriate to select the cut-off frequency of the stationary filter to be greater than the corresponding value of the real system for the convergence of the POE algorithm. The convergence is achieved within a range of cut-off frequencies.

Keywords: Pseudo-Output Error Algorithm, Stationary Filter, Gauss-Newton, System Identification, Parameter Estimation.

Presenting Author's biography

Antoine Abche. He received the BS degree and MS degree in Electrical Engineering from the University of Toledo (USA) in 1984 and 1986, respectively. He received the PhD degree in Biomedical Engineering from Rutgers The State University (USA) in conjunction with the University of Medicine and Dentistry of New Jersey in 1996. Currently, he is a professor in the department of electrical Engineering at the University of Balamand. Dr Abche's research interests are: Virtual Reality, DSP, Image Processing, Analysis and Classification, Telemedicine, Image Registration, Neural Network, Fuzzy Logic, Modeling and System Identification.

1 Introduction

One aspect of system identification is the parameters' estimation of an assumed model that best characterizes the real system to be identified. System identification algorithms can be classified generally into two categories: i) the Equation Error (EE) algorithms and ii) the Output Error (OE) Algorithms [1]. The mechanism, the accuracy and the ability to converge to the desired solution differ from one algorithm to another.

The convergence of optimization algorithms to global or to local extremums has been a concern of great importance to researchers using such algorithms in their respective fields, particularly, in system and/or model identification. The global convergence is generally attained by using assumptions that are not always valid in practice, requiring complicated mathematical developments or time consuming computations. The literature is filled with methods of optimization for a wide variety of applications. Generally, they can be classified in two main categories: i) The derivative methods and ii) the non-derivative methods.

The derivative methods are based on the computation of the derivative of a cost or objective function to determine the direction of the search and the values of the parameters to be identified for the next iteration. The algorithms that are well known and most efficient include Newton/Gauss-Newton, Levenberg-Marquardt, conjugate gradient, Quasi-Newton. They are characterized by a rapid convergence near the minimum. The main disadvantages include the sensitivity to the initial conditions (such as the values of the unknown parameters) and the convergence to secondary minimums.

On the other hand, the non derivative methods do not require the computation of the derivative of the cost function. Consequently, they are not attracted to secondary minimums during the convergence procedure. It needs only evaluations of the function to be minimized. This represents a major advantage. They include: the heuristic methods that explore the space by successive iteration (simplex), and the stochastic methods that explore points of space by following a random process (Monte-Carlo, simulated annealing, genetic). The drawbacks include a slow convergence and they often require a fine adjustment or tuning of the parameters.

Generally, the derivative algorithms are somewhat more powerful than the approaches requiring only the evaluation of the function to be minimized. But, it is not always enough to compensate for the computation of the derivatives every iteration. However, it can be found applications in which a derivative-based approach performs much better than a non-derivative based approach and vice-versa. The algorithms presented in this paper can be classified among the derivative methods.

The Output Error algorithms for system' identification offer asymptotically a non biased estimation [2], [3]. Unfortunately, this property is achieved at the cost of the minimization of a quadratic criterion by Non Linear Programming (NLP). This is generally translated into a non uniqueness of the optimum [4]. This fundamental problem can be solved by a pre-optimization procedure. Some researchers propose an optimization technique based on a genetic algorithm [5], even though it has its limitation. Thus, by initializing an optimization algorithm of the Newton's type near the global optimum, one can benefit from the presented properties. However, this procedure is costly in terms of computation time.

Other researchers offer a pre-initialization procedure that is based on EE algorithms i.e. a linear optimization procedure using a Least Square approach [1], [6], [7], [8]. Even though the first estimation is asymptotically biased, it is generally close to the global optimum. This methodology can be tuned and perfected by a variable metric technique.

The problem of convergence of Output-Error algorithms can be resolved by approaching the problem from a different perspective. In this context, a researcher has implemented a recursive algorithm based on the theory of Hyper-stability [9]. The analysis shows that it converges to a unique optimum. The apparent contradiction with the general approach is due to the fact that this recursive algorithm does not minimize a quadratic criterion. In this regard, an off line algorithm is proposed to generalize the precedent approach [10], [11]. The algorithm is called the Pseudo-Output Error (POE) Algorithm [12].

The POE algorithm is based on the introduction of a stationary filter in the computation of the sensitivity functions in order to improve the convergence to a global optimum and to facilitate the use of the Output Error (OE) algorithms of the Newton's type as an initialization step. However, the selection of the filter is highly crucial in achieving the convergence toward the desired solution and consequently the identification of the system under study. Thus, the appropriate filter's selection requires a priori information (such as the Pass band) about the real system. In this context, the effect of the filter's cut-off frequency on the POE algorithm is investigated, analyzed and quantitatively evaluated. The evaluation is performed using Monte-Carlo simulation techniques.

This paper is organized as follows: A presentation of the system's model and the POE algorithm are given in Section 2. The method of evaluation is presented in Section 3. The performed Monte-Carlo experiments and the corresponding results and analysis are presented in Section 4. Finally, a conclusion is presented in Section 5.

2 Method

2.1 The real System

Consider a stable continuous time system defined by

$$y(t) = \frac{B(q)}{A(q)} u(t) + w(t) \qquad (1)$$

Where

"q" is the derivative operator

y(t) is the output response of the system

u(t) is the input signal to the system

w(t) is the added noise that perturbs the output response

The functions A(q) and B(q) are defined as follows:

$$A(q) = 1 + a_1 q + \cdots\cdots + a_{n-1} q^{n-1} + a_n\ q^n \qquad (2)$$

$$B(q) = b_0 + b_1\ q + \cdots\cdots + b_m\ q^m\ , \quad m < n$$

That is, Eq. (1) can be expressed as

$$\begin{aligned} &y(t) + a_1 y'(t) + a_2 y''(t) + \cdots + y^{(n)}(t) = \\ &\quad b_0 u(t) + b_1 u'(t) + \cdots + b_m u^{(m)}(t) + w(t) \end{aligned} \qquad (3)$$

2.2 The model

As already stated, the problem of system identification involves the estimation of the parameters of the system. In this case, they are the "a_1,…a_n" and "b_0….b_m". Also, it involves the definition of the model since that indicates the number of unknowns to be estimated. Thus, the structure of the system is assumed to be known and the model to be identified can be expressed as

$$y_M(t, \hat{\theta}) = \frac{\hat{B}(q, \hat{\theta})}{\hat{A}(q, \hat{\theta})} u(t) \qquad (5)$$

Where $y_M(t, \hat{\theta})$ is the output response of the model due to the input excitation u(t) and $\hat{\theta}$ represents the parameters of the system to be identified. The latter vector is given by

$$\hat{\theta}^T = [\hat{a}_1, \hat{a}_2, \cdots\cdots, \hat{a}_n, \hat{b}_0, \cdots\cdots, \hat{b}_m] \qquad (6)$$

The output of the model at the i^{th} iteration can be expressed as

$$y_M(t, \hat{\theta}) = \hat{\theta}_i^T\ \varphi_M(t, \hat{\theta}_i)$$

Where

$$\begin{aligned} \varphi_M(t, \hat{\theta}_i) = [&-q\ y_M(t, \hat{\theta}_i), \cdots, -q^n y_M(t, \hat{\theta}_i), \\ &u(t), q\ u(t), \cdots, q^m\ u(t)]^T \end{aligned}$$

2.3 The POE Algorithm

The objective of system identification is to estimate the parameters of the model such that the model's output $y_M(t, \hat{\theta})$ is close as possible to the measured noisy output $y(t)$. Therefore, the estimation is accomplished by forming a quadratic criterion function $J(\theta)$ defined in terms of the difference between the output response of the model and the collected noisy output signal i.e.

$$J(\theta) = \frac{1}{N} \sum_{i=1}^{N} (y(t) - y_M(t, \hat{\theta}))^2 \qquad (7)$$

$$= \frac{1}{N} \sum_{i=1}^{N} (\varepsilon_M(t))^2$$

where N is the number of the measurements and $\varepsilon_M(t)$ is the error signal.

Therefore, the above function $J(\theta)$ must be minimized to estimate the optimal parameters θ^* of the assumed model such that $y_M(t, \theta^*)$ will best approximate the measured output signal.

The minimization is performed using an iterative non-linear optimization technique, namely, the Gauss-Newton algorithm. The update of the parameters from iteration to another is given by

$$\hat{\theta}_{(i+1)} = \hat{\theta}_i - \left\{ \left[J''_{\hat{\theta}} \right]^{-1} J'_{\hat{\theta}} \right\}_{\hat{\theta} = \hat{\theta}_i} \qquad (8)$$

Where the gradient $J'_{\hat{\theta}}$ and the Hessian $J''_{\hat{\theta}}$ are computed in terms of the sensitivity function as

$$J'_{\hat{\theta}} = \frac{\partial J}{\partial \hat{\theta}} = -\frac{2}{N} \sum \sigma(t, \hat{\theta})\ \varepsilon_M(t) \qquad (9)$$

$$J''_{\hat{\theta}} = \frac{\partial^2 J}{\partial \hat{\theta}\ \partial \hat{\theta}^T} \approx \frac{2}{N} \sum_{t=1}^{N} \sigma(t, \hat{\theta})\ \sigma^T(t, \hat{\theta}) \qquad (10)$$

The sensitivity functions are given by

$$\sigma(t, \hat{\theta}) = \frac{\partial y_M(t, \hat{\theta})}{\partial \hat{\theta}} \qquad (11)$$

That is,

$$\sigma_{\hat{a}_j}(t, \hat{\theta}_i) = \frac{-q^j}{\hat{A}(q, \hat{\theta})} y_M(t, \hat{\theta}_i) \qquad (12)$$

$$\sigma_{\hat{b}_j}(t, \hat{\theta}_i) = \frac{q^j}{\hat{A}(q, \hat{\theta})} u(t, \hat{\theta}_i) \qquad (13)$$

Subsequently, the vector of the sensitivity can be written as

$$\sigma(t,\hat{\theta}_i) = \frac{1}{\hat{A}(q,\hat{\theta}_i)}\varphi(t,\hat{\theta}_i) \qquad (14)$$

Where

$$\sigma(t,\hat{\theta}_i) = [\,\sigma_{\hat{a}_1}(t,\hat{\theta}_i), \sigma_{\hat{a}_{21}}(t,\hat{\theta}_i), \cdots, \sigma_{\hat{a}_n}(t,\hat{\theta}_i), \\ \sigma_{\hat{b}_0}(t,\hat{\theta}_i), \cdots\cdots, \sigma_{\hat{b}_m}(t,\hat{\theta}_i)\,]^T$$

The above sensitivity function depends on the filter $\hat{A}(q,\hat{\theta}_i)$ and consequently, its corresponding parameters. Having initialized the Gauss Newton algorithm by the initial parameters, the above sensitivity functions vary every iteration.

The developed POE algorithm involves the introduction of a stationary filter D(q) in which the parameters does not vary from one iteration to another. That is, the filter $\hat{A}(q,\hat{\theta}_i)$ is replaced with the filter D(q) and the Pseudo-sensitivity functions become

$$\phi(t,\hat{\theta}_i) = \frac{1}{D(q)}\varphi(t,\hat{\theta}_i) \qquad (15)$$

Thus, the POE algorithm depends on the stationary filter D (s) which should be carefully selected.

Subsequently, the Pseudo-Output Error (POE) algorithm is obtained

$$\hat{\theta}_{i+1} = \hat{\theta}_i + \left\{\sum_{i=1}^{N}\phi(t,\hat{\theta}_i)\,\phi^T(t,\hat{\theta}_i)\right\}\sum_{i=1}^{N}\phi(t,\hat{\theta}_i)\,\varepsilon_M(t,\hat{\theta}_i) \qquad (16)$$

At first, the Gauss-Newton OE algorithm (Eq. (8)) and the Pseudo-Output Error Algorithm (Eq. (16)) appear to be similar. The only difference is manifested in the computation of the sensitivity functions. Using the Gauss-Newton OE algorithm, the sensitivity functions are obtained based on the filter ($\frac{1}{\hat{A}(q,\hat{\theta}_i)}$). The corresponding parameters vary from one iteration to another. On the other hand, using the POE algorithm, the sensitivity functions (or rather pseudo-sensitivity functions) are obtained based on the filter ($\frac{1}{D(q)}$) where the parameters are fixed. This difference is fundamental in the mechanism of the convergence. Effectively, while the algorithm of Gauss-Newton OE algorithm is based on the minimization of a quadratic criterion, the POE is not i.e. the minimization does not exist. In the latter case, the convergence to secondary minimums does not exist. Therefore, the selection of the filter $\frac{1}{D(q)}$ is crucial in the behavior of the POE algorithm.

The ideal stationary filter is a filter defined by the real parameters of the system i.e. D(q)=A(q). However, the latter parameters are unknown. The introduced filter is a low pass filter and has the same order of the model. Thus, an even less accurate estimation of its pass band (i.e. bandwidth), assumed as a priori knowledge and defined in terms of its cut-off frequency, is investigated and its effect on the convergence of an OE system identification algorithm is studied and is analyzed.

3 Method of Evaluation

The dependence of the convergence of the POE algorithm on the bandwidth of the stationary filter is investigated using Monte-Carlo (MC) simulation techniques. Several scenarios were performed. Each scenario refers to a particular selection of 1/D(q). The assumed model and the true system are excited by the same input signal.

The approach can be summarized as follows: Given a real system, the output response is determined for a given input signal. Having the input and output measurements, the next step is to define the structure of the model. It is assumed to have the same structure as the real system. Then, the parameters of the model are assigned a set of initial values. At this point, the system identification procedure (such as the POE, Gauss-Newton OE algorithm,…) is performed. In other words, an optimization procedure is performed in order to estimate the desired parameters by minimizing an objective function defined in terms of the difference between the output response of the model and the output response of the real system. Thus, the unknown parameters are varied until the output response of the model is as close as possible to the output of the real system.

4 Results and Analysis

In this section, Monte-Carlo simulations are performed to study the dependence of the convergence of the POE system identification algorithm on the cut-off frequency of the stationary filter. The results of a second and third order systems are presented.

The real system to be identified is a weakly oscillating second order system. It is defined in the Laplace domain (s-domain) by the following transfer function:

$$H(s) = \frac{w_n^2}{s^2 + 2\,\xi\,w_n s + w_n^2} \qquad (17)$$

Where w_n and ξ are the natural frequency and the damping coefficient of the oscillating system, respectively. They are selected to be 1.5 rad/s and 0.001, respectively. The input excitation is a periodic square signal. The sampling period is equal to 0.1 s.

The selection of an oscillating second order system and a periodic excitation function leads to a quadratic criterion characterized by pseudo secondary minimums as illustrated in Fig. 1. Therefore, this selection will greatly show the performance of the POE algorithm when it is compared with another OE algorithm, namely, the Gauss-Newton OE algorithm, in which the filter that is used in the computation of the sensitivity functions varies from an iteration to another.

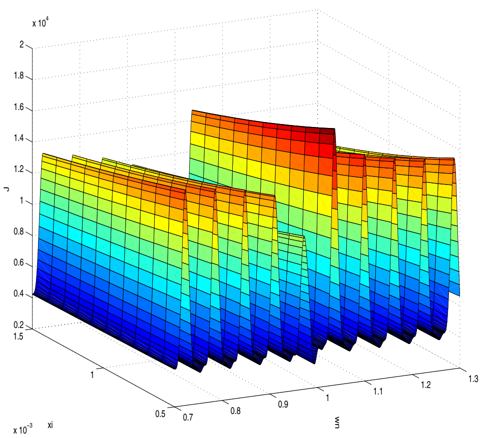

Fig. 1: the quadratic criterion $J(w_n, \xi)$ as a function of the two parameters of the model to be identified.

As already stated earlier, the objective is to estimate the two parameters w_n and ξ, and consequently, to identify the desired system. Using the POE algorithm, the pseudo-sensitivity functions are derived and are given by the following expressions

$$\sigma_{w_n}(s) = \frac{1}{D(s)}\left(\frac{2}{w_n} s^2 + 2\,\xi\, s\right) Y_M(s) \quad (18)$$

$$\sigma_{\xi}(s) = \frac{1}{D(s)}\left(-2\, w_n\, s\right) Y_M(s) \quad (19)$$

where $Y_M(s)$ is the output response $y_M(t)$ in the Laplace Domain, $\sigma_{w_n}(s)$ is the sensitivity function of the parameter w_n in the Laplace Domain, $\sigma_{\xi}(s)$ is the sensitivity function of ξ in the Laplace domain and D(s) is the stationary filter.

Then, the model's structure of the system is defined. It is assumed to have the same structure as that of the real system (i.e. assumed to be a second order system). Consequently, the selected stationary filter is a second order low pass filter defined by

$$\frac{1}{D(s)} = \frac{1}{(a\, s / b + 1)^2} \quad (20)$$

The selection of an aperiodic filter allows to show that, in spite of *a poor priori* knowledge, good results are obtained.

Having initialized the parameters to be estimated, the Pseudo-Output Error algorithm is executed to identify the desired system. Each Monte-Carlo experiment is performed using the same input/output data. The initial values of w_n are selected to be within a certain range and the damping coefficient is selected to be equal to 0.1 for all Monte-Carlo simulations.

Figures 2 and 3 illustrate the results of one MC experiment and show the performance of two identification algorithms, the POE algorithm and the Gauss-Newton OE algorithm, respectively. The experiment is performed by initializing the parameters to be estimated w_n and ξ to 4 rad/s and 0.1, respectively. Each figure shows three different subplots: the objective function to be minimized $J(w_n, \xi)$ (Upper subplot), the damping coefficient ξ (lower plot) and the natural frequency w_n (middle plot). Each variable is displayed as a function of the number of iteration. The cut-off frequency of the stationary filter is assumed to be 1.5 rad/sec.

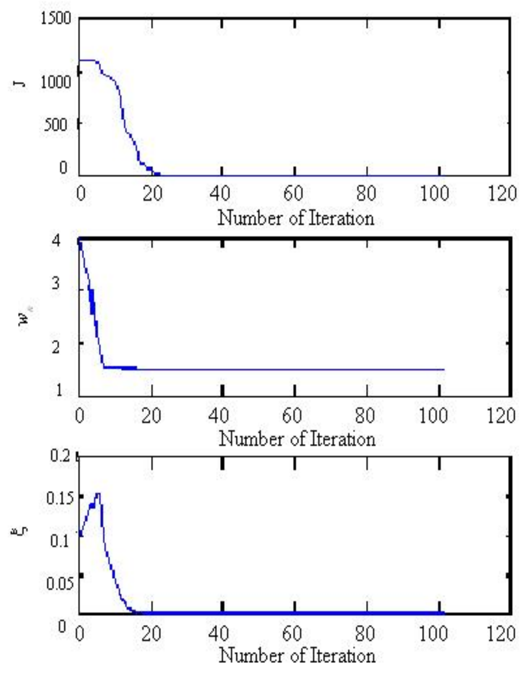

Fig. 2: The evolution of J, w_n, ξ as a function of the Number of iteration using the POE algorithm.

In this MC experiment, the results show that the POE algorithm converges to the desired parameters' values with a smaller number of iteration. On one hand, the POE algorithm satisfies the selected tolerance within 45 iterations. On the other hand, it requires more than

300 iterations for the Gauss-Newton OE algorithm. The tolerance is defined with respect to two successive values of the objective function $J(w_n, \xi)$. Second, the POE algorithm converges to the desired parameters while the OE converges to a secondary minimum as illustrated by the corresponding plots. Thus, the performance of the POE algorithm is better than the Gauss- Newton OE algorithm.

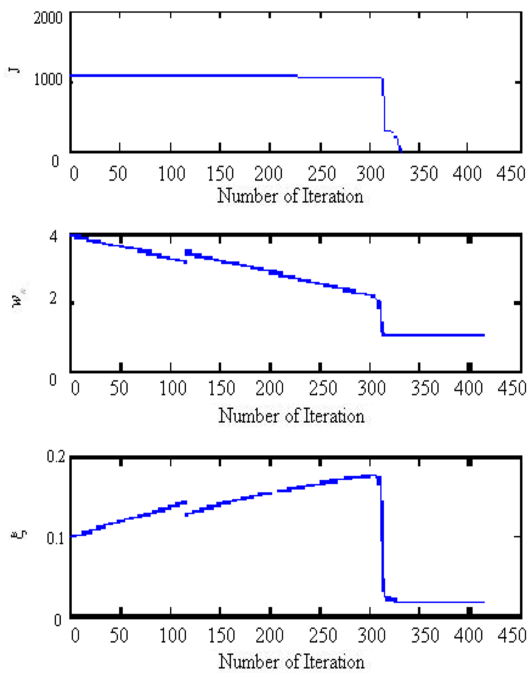

Fig. 3: The evolution of J, w_n, ξ as a function of the Number of iteration using the Gauss-Newton OE algorithm.

Table 1 displays the results of the Monte-Carlo simulation experiments in which the stationary filter is selected to be a second order filter with a cut-off frequency $w_c (= 2\pi f_c)$ of 1.5 rad/sec. It illustrates four fields: i) the initial value of w_n ii) the optimized value w_n, iii) the optimized value ξ and iv) the number of iteration. The initial value of the damping coefficient is selected to be 0.1.

The results of the table show that the POE algorithm converges to the desired values if w_n is selected within a certain range i.e. $1.4 \le w_n < 8.0 \text{ rad/sec}$. Otherwise, the POE identification algorithm converges to a secondary optimum or diverges.

Figure 4 elaborates further the effect of the bandwidth. It shows the dependence of the optimized values w_n on the cut-off frequency w_c of the stationary filters; (a) 0.5 rad/sec, (b) 1.0 rad/sec, (c) 2.0 rad/sec, 4.0 rad/sec, (e) 8.0 rad/sec and (f) 10.0 rad/sec. It is evident that the selection of the stationary filter's cut-off frequency is crucial for the convergence of the POE algorithm. The results highlight the following observations: i) it is more appropriate to select w_c greater (or equal) than the true cut-off frequency of the real system, ii) for the given initial values (ξ =0.1 & varying w_n) and a particular stationary filter, the POE algorithm can converge to the true values for a certain range of w_n iii) the range is larger when w_c is selected greater than the true value, iv) the POE algorithm converges to a secondary optimum or diverges if w_c is selected beyond a certain value (upper bound).Thus, there is a range of w_c within which the POE algorithm converges to the desired solution. Otherwise, the identification algorithm diverges.

Table 1. Performance of the POE algorithm (ξ =0.001)

Initial Value w_n	Optimized w_n	Optimized ξ	Number of iteration
<=1.3	Diverge	diverge	
1.4	1.5	0.001	343
1.5	1.5	0.001	73
1.7	1.5	0.001	31
1.9	1.5	0.001	23
2.1	1.5	0.001	22
2.5	1.5	0.001	18
3.0	1.5	0.001	26
3.5	1.5	0.001	35
4.0	1.5	0.001	45
5.0	1.5	0.001	72
6.0	1.5	0.001	144
7.0	1.5	0.1191	572
8.0	5.08	0.857	66

Similar observations can be deduced from the plots that show the dependence of the optimized ξ on w_c (Fig. 5).

Another case of a second order system is studied. The damping coefficient is chosen to be greater than 1 (i.e. ξ =1.2) and w_n = 1.5 rad/sec (true $w_c \approx 0.75$ rad/sec). In this particular case, similar conclusions can be deduced even though the range of w_n for which the POE algorithm converges, is wider (Table 2). The initial value of the damping coefficient is selected to be 0.1. The table shows the range of w_n within which the POE algorithm converges for various stationary filters characterized by their respective w_c.

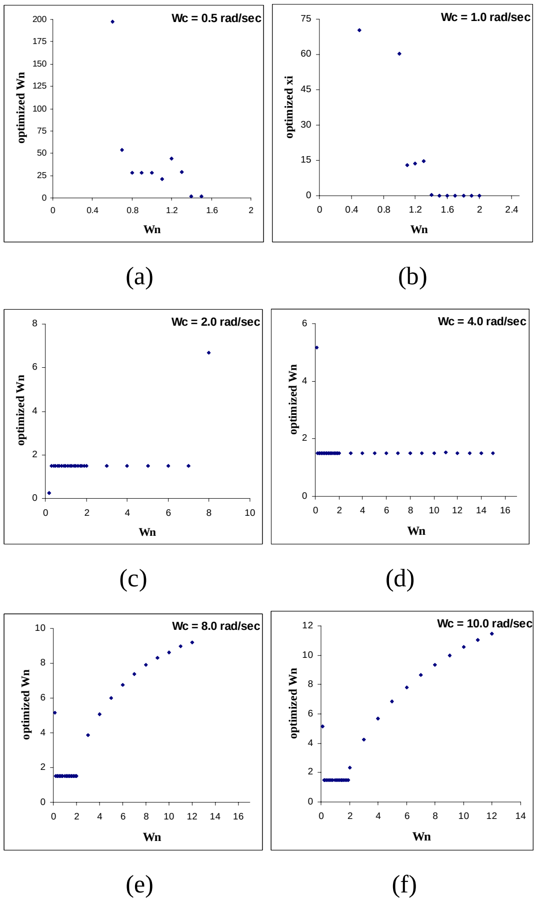

Fig. 4: The effect of the cut-off frequency w_c of the stationary filter on the optimized value w_n.

Table 2: Performance of the POE algorithm (ξ =1.2)

w_c	Range wn	w_c	Range wn
0.5	[2, 8]	5.0	[0.2, 20]
1.0	[0.2, 15]	8.0	[0.2, 3]
1.5	[0.2, 20]	10.	Divergence
3.0	[0.2, 30]	15	Divergence

Also,, the Monte-Carlo simulations are performed on a third order system that is defined by the following transfer function:

$$H(s) = \frac{b_0}{s^3 + a_2 s^2 + a_1 s + a_0} \qquad (21)$$

where $a_0 = 1$, $a_1 = 2.2$, $a_2 = 2.2$, $b_0 = 1$.

The stationary filter is assumed to be

$$\frac{1}{D(s)} = \frac{1}{(a\, s / b + 1)^3} \qquad (22)$$

The true cut-off frequency of the real system is about 0.88 rad/sec. The system is excited by a periodic square signal. The sampling period is equal to 0.1 s.

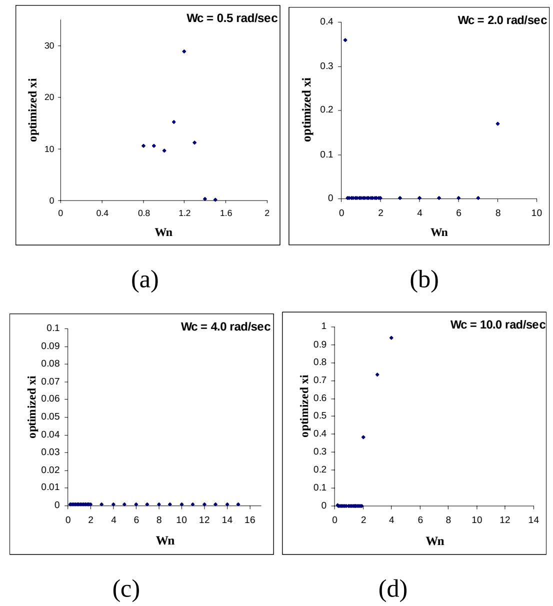

Fig. 5: The effect of the cut-off frequency w_c of the stationary filter on the optimized value ξ.

The MC experiments are performed by selecting different stationary filters characterized by different cut-off frequencies. The POE algorithm is executed using the following initial values $a_0 = 2$, $a_1 = 2$, $a_2 = 3$, $b_0 = 3$. Fig. 6 shows typical results of the convergence of the POE to the desired values. The cut-off frequency of the stationary filter is selected to be 3.0 rad/sec. On the other hand, the OE system identification algorithm does not converge with these set of initial values of the parameters.

A summary of the results are tabulated in Table 3. It displays the optimized values of the POE algorithm as a function of w_c of the selected stationary filter. It is evident that for the given initial values i) the POE identification algorithm can converge to the desired values when the cut-off frequency of the stationary filter is greater than the real value i.e. w_c >0.88 rad/sec, ii) the Convergence is achieved for a range of w_c i.e. 0.9 rad/sec <= w_c < 8 rad/sec.

5 Conclusion

In this paper, the effect of the bandwidth of the stationary filter on the convergence of the POE algorithm is investigated. The evaluation is performed using Monte-Carlo simulation techniques. The latter filter is introduced in the POE algorithm to compute the corresponding sensitivity functions of the well-known Gauss-Newton OE algorithm. These functions are not the true functions of sensitivity. Therefore, the name of pseudo-functions of sensitivity and the

Gauss-Newton OE algorithm becomes the Pseudo-Output Error (POE) algorithm.

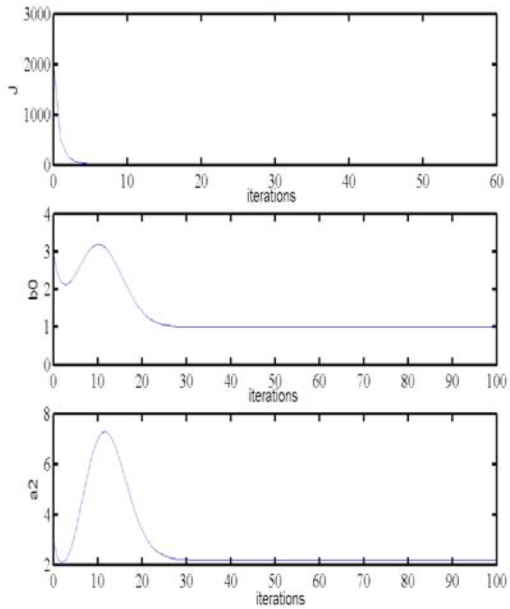

Fig. 6: The evolution of J, b_0, a_2 as a function of the Number of iteration using the POE algorithm.

Table 3: The performance of the POE algorithm: (3rd order system with the desired cut-off frequency ≈ 0.88 rad/sec)

w_c	b_0	a_2	a_1	a_0
< 0.7	Div.	Div.	Div.	Div.
0.7	-0.84	-3.38	1.5	-0.23
0.8	-0.53	1.52	-1.9	-0.49
0.9	1	2.2	2.2	1
1	1	2.2	2.2	1
1.5	1	2.2	2.2	1
3.0	1	2.2	2.2	1
4.0	1	2.2	2.2	1
5.0	1	2.2	2.2	1
6.0	1	2.2	2.2	1
7.0	1	2.2	2.2	1
8.0	Div.	Div.	Div.	Div.

The simulation results show that the filter's cut-off frequency should be selected higher than the true value. However, it should not be selected far from F_C. The appropriate selection of the stationary filter has lead to the convergence of the POE algorithm to the desired values. Otherwise, the identification algorithm diverges and consequently, the corresponding system is not identified. Even though *a priori* knowledge is poor, good results of convergence are obtained with the POE algorithm.

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