HIGHER-ORDER DIFFERENTIAL DELAY SYSTEMS WITH CONTROL APPLICATIONS

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Abstract

We are interested in solving and presenting some applications of higher order linear descriptor differential systems given by the expression:

$$FX^{(r)}(t) = GX(t-\tau),\ \tau > 0$$

with constant square coefficients and consistent initial conditions. Higher order linear descriptor systems can result from several types of linearization of general non-linear high order descriptor delay differential systems of the general form: $F\left(\underline{x}, \underline{\dot{x}}, \ldots, \underline{x}^{(n)}\right) = 0$. Typical applications where second order descriptor systems naturally arise involve multi-body systems and networked control systems (NCS). In our case, in order to solve such kind of systems, we apply the complex Weierstrass canonical form (WCF) and the Drazin inverse theory. Indeed, these two effective tools for the solution of descriptor systems have been systematically used in different areas of control and systems theory. Applying the WCF, two lower dimension sub-systems are obtained with a particular structure. A numerical example from the emerging area of NCS with constant and unknown network induced delays is presented as our basic motivation.

Keywords: Descriptor Systems, Time Delay Systems, Networked Systems

Presenting Author's biography

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1. Introduction

In the theory of system science (with significant applications to engineering), it is valid to claim that the characteristics and nature of the process of synthesis and global instrumentation depend on the *type* of available models, see for more details [18].

Thus, there are models where some of the internal variables are classified into potential inputs, outputs, internal variables and referred to as *oriented* models, or models where no classification has been made of the internal variables and are called *implicit* models. All such models may be used for the selection of effective sets of inputs and outputs, and they are referred to as progenitor models. Additionally they may be classified as:

(a) Internal Models,

(b) External Models, and

(c) Internal–External Models.

In our case, we are interested in *internal* models. These models, see [20], have a long history and they are primarily described in terms of *first* order ordinary nonlinear equations and they are the standard state-space descriptions of the implicit type

$$F\left(\underline{x},\dot{\underline{x}}\right)=0\,, \tag{1}$$

where $\underline{x}$ is the vector of all internal model variables. For the linear case, the above system can be reduced to

$$E\dot{\underline{x}}=A\underline{x}\,. \tag{2}$$

When the inputs $\underline{u}$ and outputs $\underline{y}$ have been defined, the non-linear control model is defined by

$$F\left(\underline{x},\dot{\underline{x}},\underline{u}\right)=0\,,\ \ \underline{y}=G\left(\underline{x},\dot{\underline{x}},\underline{u}\right), \tag{3}$$

and for the linear case is expressed by

$$E\dot{\underline{x}}=A\underline{x}+B\underline{u}\,,\ \ \underline{y}=C\underline{x}+D\dot{\underline{x}}+E\underline{u}\,. \tag{4}$$

In the existing literature, linear internal models are called Descriptor (differential/difference) systems (or generalized state pace systems or differential algebraic systems), and they have a key role in the modelling and simulation process of constrained dynamical systems.

These type of systems have gained significant popularity in the past years since in many real situations are capable of providing a complete description of dynamical systems than the traditional state space modelling set up see [10], and [31]. For a more systematic and comprehensive exposition of the most significant aspects regarding the theory, the numerical treatments and various applications see [6-8], [10], [14-15], [18-21], [23], [27] and the references therein.

Quite often and in several applications, we are faced with systems of higher order, i.e.

$$F\left(\underline{x},\dot{\underline{x}},\ldots,\underline{x}^{(n)}\right)=0\,, \tag{5}$$

Then autoregressive descriptions are used, see [26].

Moreover, *linear higher order differential (descriptor)* systems can arise from several types of linearization of a general non-linear high order differential (descriptor) system, see [21].

Typical applications where second order descriptor systems naturally arise are multi-body systems; see [10] and [23]. To motivate and justify further the analysis and the importance of our approach, we present the following classical example.

Example 1 The mathematical *pendulum*

$$\begin{aligned} m\ddot{x}+2\lambda x&=0,\\ m\ddot{y}+2\lambda y+mg&=0, \end{aligned} \tag{6}$$

in the case of a multi-body system, see for more details [3] and [14].

In this conference paper, two main directions are presented and discussed. At first, we want to provide an engineering motivation about delay systems. Especially, we will focus on Networked Control Systems. Some simulation results are also presented and commented. As a second direction, we will try to give a brief description of the mathematical extensions of higher order descriptor differential systems. Some elements of matrix pencil and Drazin inverse theory are also presented. Several interesting open issues for further research are considered.

In the next section, we provide the engineering motivation for the time delay systems.

2. Engineering Motivations for time delay systems

The issue of time-delay is of great importance in various areas of control technologies and instrumentation such as power systems, industrial process control including the steel and oil industry, machining and metallurgical processes, remotely operated robots and control over computer networks (or as it is also known *Networked Control Systems*) to name a few; see [30]. The last two mentioned engineering disciplines deserve some more analysis as they present a potential field of application for the theory developed in this article.

A Networked Control System (NCS) is a feedback control system where the feedback loops are closed by means of an electronic network [4] and [16]. It is well known that Networked (Control) Systems are not subject to the same design assumptions as non-networked systems, a fact that is mainly due to the inevitable presence of network delays and packet drops.

In a typical closed-loop NCS, the state is sampled periodically, transmitted through the network, becomes available to the controller, which after computing the control action, transmits the sampled signal to the event-driven actuator after an uncertain or constant (but unknown) delay. The plant receives this command via a ***Zero Order Hold*** device (ZOH) after a delay τ, which models the sum total of the involved transmission delays. These network-induced delays appear in the information flow between the sensor and the controller (delay $\tau_{sc}(k)$), as well as between the controller and the actuator (delay $\tau_{ca}(k)$), where k denotes the dependence on the k^{th} sampling period.

Although various system-theoretic analysis tools have been used for the modelling of NCS's with delays, the most successful ones are sophisticated adaptations of analogous results from the mature area of Time Delayed Systems (TDS).

Typical examples of this "Time-Delayed" approach to NCS analysis and synthesis appear in [28, 29] where the main result is the design of a robust state feedback control law which (under some rather mild assumptions) takes care of the uncertain network-induced delays and the data packet dropout during transmission. It is interesting that in both papers the assumed structure of the memoryless controller is of the form,

$$\underline{u}(t) = -K\underline{x}(t-\tau)\,, \tag{7}$$

whereas the main synthesis tool is a carefully selected *Lyapunov-Krasovskii* functional.

Another research discipline where the presence of (network-induced) time-delays is of utmost importance for both system analysis and control synthesis is tele-operation and bilateral (master/slave) tele - robotics.

It is an experimental fact that delays of the order of several hundred milliseconds can lead to instabilities of tele-operation systems, whereas in a bilateral robot system a delay of twenty milliseconds causes significant deterioration in the time response, see for instance [9], [24] and [25].

For example in [24, 25], standard controllers (whose design was based on the delay-free system) are modified for tele-operation over an IP network and experimental results are presented on a mobile robot following a prescribed trajectory.

In [9] a typical bilateral *Tele-robotics* application is presented where a remote *slave* robot tracks the motion of a *master* robot that, in turn, is commanded by a human operator. In such a scheme, the force feedback from the slave to the master, suffers from varying delays and packet losses due to (wireless) network congestion, bandwidth, or distance. It is then shown that unless these network effects are taken into account, the stability and performance of the system may severely be degraded.

It is noted that for the majority of the above tele-operation applications, the process modelling is based on scattering transformation and passivity concepts. The modelling approach proposed in this paper possibly offers an alternative viable option, which remains to be examined in the future.

Motivated by practical and realistic problems that occur in the control of electromechanical systems, a systematic study of delayed feedback has been a field of recent research analysis and synthesis results. This is another area where the proposed modelling is useful.

At this point, we present a numerical example concerning a simplified NCS with delay. In fact, this serves as our main motivation for the present article.

The specific example is inspired by [24, 25], where a networked DC motor is controlled via a PI controller. The (open–loop stable) DC motor dynamics with armature voltage as input and angular speed (rad/s) as output are described by the transfer function

$$G(s) = \frac{2030}{s^2 + 28.59s + 60.36} = \frac{2030}{(s+26.29)(s+2.29)} \tag{8}$$

with state space description:

$$\underline{\dot{x}}(t) = \begin{bmatrix} -1.1103 & 345.0704 \\ -0.0865 & -27.4706 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 5.8824 \end{bmatrix} \underline{u}(t),$$

$$\underline{y}(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x}(t)\,, \tag{9}$$

with $\underline{x_1}(t) = \underline{y}(t)$ being the angular speed (rad/s) and $\underline{x_2}(t)$ being the armature (rotor winding) current.

The initial condition for the discretized version of the NCS is

$$\begin{bmatrix} 0 & 0 \end{bmatrix}^T. \tag{10}$$

The above dynamics are derived using the well-known DC motor dynamics with realistic values for the Moment of Inertia (J = 42.6 x 10^{-6} Kg*m^2), Inductance (L = 170x10^{-3} H), Resistance (R = 4.67 Ω) Torque and Back–EMF constants (14.7x10^{-3} V*s/rad) and Damping Coefficient (B = 47.3x10^{-6} Nm*s/rad). Note that this is an extremely "benevolent" stable minimum-phase system with infinite gain margin and a phase margin of 72 degrees.

In [24, 25], the motor's PI speed–controller is designed and tuned for a step function reference speed without concern for the network delays. The networked version of the system is examined with a sampling period $h = 1$ second. The (constant or uncertain) delay τ varies between the bounds $\tau_{min} = 0$ and $\tau_{max} = h$ seconds.

Neglecting the effect of sampling, the network presence is modelled as a lumped input delay

$$\tau = \tau_{sc}(k) + \tau_{ca}(k) \tag{11}$$

For a particular class of *scheduled* networks, this delay is practically constant. This class includes the Token Passing (TP) protocol with typical examples the token bus (IEEE Standard 802.4) and token ring (IEEE Standard 802.5).

In the provided simulations, we examine the effect of constant delay τ on the performance of a linear set-point tracking controller designed via standard LQR theory. It should be noted that the design of this controller was based on the delay-free system and consists of two parts: a static state feedback part (-Kx(t)) a fixed feed forward part Fr(t) with r(t) being the command signal.

Exploiting the LMI-based procedure presented in [32], we can compute the maximum delay value, which guarantees the stability of the delayed system for a given LQR feedback gain K computed as stated previously.

Selecting

$$Q=1*I_2 \text{ and } R=1, \tag{12}$$

the controller gains are:

$$K = \begin{bmatrix} 0.9485 & 6.9097 \end{bmatrix}, \tag{13}$$

$$F = 1.0004 \tag{14}$$

For this value of K the approach of [32] (a sufficient condition) yields a delay value of

$$\tau = \tau_{sc}(k) + \tau_{ca}(k) = 0.02 \text{ sec.} \tag{15}$$

A command signal consisting of a staircase function corresponding to a speed command varying in the range of 1000 rpm (= 104.71 rad/sec), 500 rpm (= 52rad/sec) and finally into 200 rpm (= 21 rad/sec).

For the delay-free case, the proposed controller provides perfect output tracking with no overshoot. Introducing a lumped constant delay of 0.025sec, which is slightly larger than the previous delay τ, ***Figure 1*** below shows a drastic degradation of performance of the closed loop system.

This degradation consists of an 80% overshoot and violent control action ("chattering") as shown in ***Figure 2***.

The black dotted line is the reference signal $r(t)$ to be tracked, the thick blue line $y(t)$ is the output of the non-networked (delay-free) system and the red line $y_{NCS}(t)$ corresponds to the output of the networked system suffering from constant delays.

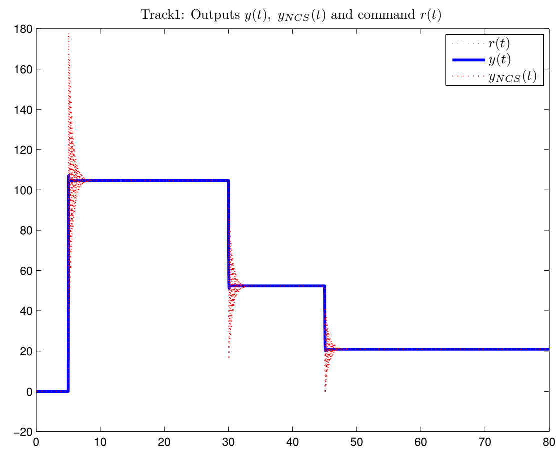

Fig. 1 Comparing the tracking performance for the delay-free and the delayed case**.**

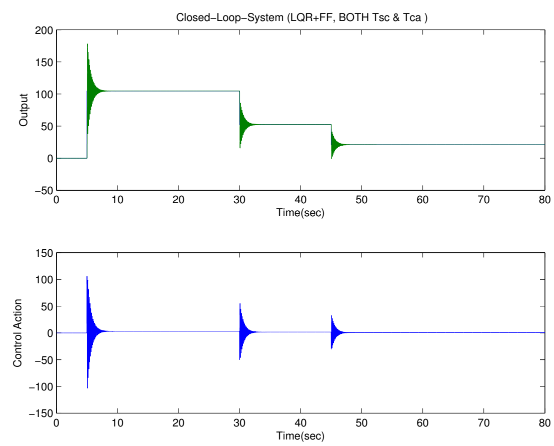

Fig. 2 Output and Control signals for a delay of 0.025sec

In the next section, several mathematical problems are presented and discussed.

3. Mathematical Extensions

In the previous section, some engineering problems with simulation results have been presented for NCS. In this section, we attempt to describe the mathematical extensions and problems that naturally arise when we want to handle/solve such type of systems. As a first extension of the existing literature, we will consider the more general case, where the state parameter is a matrix, see eqs. (1) – (3) and (5).

Considering this extension, we permit some kind of interactions between the parameters that become involved. For instance, we can have

$$X(t)=\begin{bmatrix} \underline{x}_{11}(t) & \underline{x}_{12}(t) \\ \underline{x}_{21}(t) & \underline{x}_{22}(t) \end{bmatrix}, \tag{16}$$

where with $\underline{x}_{ij}(t)$ for $i,j=1,2$, somehow we permit an (energy) interaction from NCS no.1 (i.e. $i=1,2$) to another NCS no.2. Consequently, it is clear that the vector is only a special case of the matrix case.

Furthermore, as we have also discussed in the 1st section, the more general case of higher-order differential systems is considered. Actually, if we assume that $r=1$, the classical first order case is derived.

Thus, in the system science literature, [2], [5] and [26] the generalization of the above system is given by

$$X^{(r)}(t)=AX(t), \tag{17}$$

where X is a matrix function, and is known as the standard form of *linear higher-order matrix differential equations*.

The matrix equation (17) can be treated by several well-established methods, see for instance [2], [5] and references therein. Additionally, section 5 of [26] describes a method for solving higher order equations of the form

$$q(D)X(t)=AX(t), \tag{18}$$

where q is a scalar polynomial, D is differentiation with respect to time and A is a square matrix.

Recently, as an extension of (17), the descriptor version, see (19), has been studied, see for instance [17] and [22], i.e.

$$FX^{(r)}(t)=GX(t). \tag{19}$$

However, as we have seen in the 2nd section, in many realistic engineering applications, it is consistent to design the state of the model by taking into account delays, see [1], [11-13].

In this section, our long-term purpose is to introduce and study the solution of higher order linear (homogeneous) descriptor differential systems of type (22) which is provided below and can be derived combining (20) and (21), i.e.

$$FX^{(r)}(t)=EX(t-\tau)+BU(t), \tag{20}$$

with a delayed state feedback

$$U(t)=-KX(t-\tau). \tag{21}$$

So, we obtain

$$FX^{(r)}(t)=GX(t-\tau), \tag{22}$$

where τ is a constant parameter, and X is the state matrix function. Throughout the paper, F and $G=E-BK$ are square matrices without any particular structure and the pencil $sF-G$ is regular.

At this point, it is natural to mention some additional possible extensions of the present context. It would be very interesting to consider matrices of a special structure, such as, positive (or negative!), non - negative, symmetric, skew symmetric etc, see [22].

4. Preliminary Background and Results

In this section, the preliminary background and some introductive results are given considering two well-known techniques for the solution of descriptor systems; the matrix pencil theory (i.e. complex Weierstrass canonical form) and the Drazin inverse theory.

The higher order linear descriptor delay differential systems, i.e. $FX^{(r)}(t)=GX(t-\tau)$,with $\tau>0$, and

$$\begin{cases} X(t)=\Phi_o(t), \\ X'(t)=\Phi_1(t), \\ .... \\ X^{(r-1)}(t)=\Phi_{r-1}(t), \end{cases} \quad , \ t\in[t_o-\tau,t_o] \tag{23}$$

are introduced, where $F,G\in\mathcal{M}(n\times n;\mathbb{F})$ (i.e. $\mathcal{M}$ is the algebra of $n\times n$ matrices with elements in the field $\mathbb{F}=\mathbb{C}$ or $\mathbb{R}$) with $\det F=0$, where 0 is the zero element of $\mathcal{M}(n=1,\mathbb{F})$, and $X(t)$ and $\Phi_i(t)\in \mathcal{C}^\infty(\mathbb{F},\mathcal{M}(n\times n;\mathbb{F}))$ for $i=0,1,2,\ ...,r-1$.

For the sake of simplicity, we set in the sequel $\mathcal{M}_n=\mathcal{M}(n\times n;\mathbb{F})$ and $\mathcal{M}_{m,n}=\mathcal{M}(m\times n;\mathbb{F})$.

In the sequel, some preliminary results, basic concepts and definitions concerning matrix pencil theory are introduced.

Definition 1 The pencil $sF-G$ is said to be *strictly equivalent* to the pencil $s\tilde{F}-\tilde{G}$ if and only if there exist non-singular P, $Q\in\mathcal{M}_m$ such as

$$P(sF-G)Q=s\tilde{F}-\tilde{G}. \tag{24}$$

In this work, we consider the case where that pencil is *regular*. Thus, the strict equivalence relation can be defined rigorously on the set of regular pencils as follows. Here, we regard (22) as the set of pair of non-singular elements of $\mathcal{M}_n$

The class of $\mathcal{E}_{s-e}(sF-G)$ is characterized by a uniquely defined element, known as a complex Weierstrass canonical form, sF_w-Q_w, specified by the complete set of invariants of $\mathcal{E}_{s-e}(sF-G)$. This is the set of *elementary divisors* (e.d.) obtained by factorizing the invariant polynomials $f_i(s,\hat{s})$ into powers of homogeneous polynomials irreducible over the field

$\mathbb{F}$. In the case where $sF-G$ is a regular, we have e.d. of the following type:

- e.d. of the type s^p are called *zero finite elementary divisors* (**z. F.e.d.**)
- e.d. of the type $(s-a)^{\pi}$, $a \neq 0$ are called *nonzero finite elementary divisors* (**nz. F.e.d.**)
- e.d. of the type $\hat{s}^q$ are called *infinite elementary divisors* (**i.e.d**.).

Let $B_1, B_2, ..., B_n$ be elements of $\mathcal{M}_n$. The direct sum of these elements denoted by $B_1 \oplus B_2 \oplus ... \oplus B_n$ is the *block diag* $\{B_1, B_2, ..., B_n\}$.

Then, the complex Weierstrass form $sF_w - Q_w$ of the regular pencil $sF-G$ is defined by

$$sF_w - Q_w \triangleq sI_p - J_p \oplus sH_q - I_q, \tag{25}$$

where the first normal Jordan type element is uniquely defined by the set of f.e.d.

$$(s-a_1)^{p_1}, \dots, (s-a_\nu)^{p_\nu}, \ \sum\nolimits_{j=1}^{\nu} p_j = p \tag{26}$$

of $sF-G$ and has the form

$$sI_p - J_p \triangleq sI_{p_1} - J_{p_1}(a_1) \oplus \dots \oplus sI_{p_\nu} - J_{p_\nu}(a_\nu). \tag{27}$$

And also the q blocks of the second uniquely defined block $sH_q - I_q$ correspond to the i.e.d.

$$\hat{s}^{q_1}, \dots, \hat{s}^{q_\sigma}, \ \sum\nolimits_{j=1}^{\sigma} q_j = q, \tag{28}$$

of $sF-G$ and has the form

$$sH_q - I_q \triangleq sH_{q_1} - I_{q_1} \oplus \dots \oplus sH_{q_\sigma} - I_{q_\sigma}. \tag{29}$$

Thus H_q is a nilpotent element of $\mathcal{M}_n$ with index $q^* = \max\{q_j : j = 1, 2, \dots, \sigma\}$, where

$$H_q^{q^*} = \mathbb{O}, \tag{30}$$

and $I_{p_j}, J_{p_j}(a_j), H_{q_j}$ are defined as

$$I_{p_j} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \in \mathcal{M}_{p_j}, \quad J_{p_j}(a_j) = \begin{bmatrix} a_j & 1 & 0 & \cdots & 0 \\ 0 & a_j & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & a_j & 1 \\ 0 & 0 & 0 & 0 & a_j \end{bmatrix} \in \mathcal{M}_{p_j} \tag{31}$$

and

$$H_{q_j} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \in \mathcal{M}_{q_j}. \tag{32}$$

From the regularity of $sF-G$, there exist non singular matrices P and Q such that

$$PFQ = F_w = I_p \oplus H_q \tag{33}$$

$$PGQ = G_w = J_p \oplus I_q, \tag{34}$$

where I_p, J_p, H_q are given by (31) and (32).

For the second approach, i.e. using Drazin inverses, the interested reader can be advised by [7-8]. Here we present only some preliminary steps.

Definition 2 The *Drazin inverse* of square matrix $A \in \mathcal{M}_n$, $Ind(A) = \nu$ is the matrix A^D satisfying

(i) $A^D A A^D = A^D$,

(ii) $AA^D = A^D A$,

(iii) $A^{k+1} A^D = A^k$,

for $k \geq \nu = Ind(A)$.

Definition 3 Define the matrices $\hat{F}_\lambda \triangleq (\lambda F + G)^{-1} F$ and $\hat{G}_\lambda \triangleq (\lambda F + G)^{-1} G$, for $\lambda \in \mathbb{C}$ and inverse matrix $(\lambda F + G)^{-1}$, where F, $G \in \mathcal{M}_n$.

According to Definition 3, there always exists $\lambda \in \mathbb{C}$ such that the matrix $(\lambda F + G)^{-1}$ is invertible, where F, $G \in \mathcal{M}_n$. Now, multiplying system (22) by left with $(\lambda F + G)^{-1}$ (analogously, system (19) or (20)), we obtain

$$(\lambda F + G)^{-1} F X^{(r)}(t) = (\lambda F + G)^{-1} G X(t)$$
$$\Leftrightarrow \hat{F}_\lambda X^{(r)}(t) = \hat{G}_\lambda X(t)$$

Hence, according to expressions (9) and (10), we can take

$$\begin{aligned} \hat{F}_\lambda &= (\lambda F + G)^{-1} F \\ &= \left(\lambda P^{-1} F_w Q^{-1} + P^{-1} G_w Q^{-1}\right)^{-1} P^{-1} F_w Q^{-1} \\ &= Q(\lambda F_w + G_w)^{-1} F_w Q^{-1} \end{aligned} \tag{35}$$

and

$$\begin{aligned} \hat{G}_\lambda &= (\lambda F + G)^{-1} G \\ &= \left(\lambda P^{-1} F_w Q^{-1} + P^{-1} G_w Q^{-1}\right)^{-1} P^{-1} G_w Q^{-1} \\ &= Q(\lambda F_w + G_w)^{-1} G_w Q^{-1} \end{aligned} \tag{36}$$

The above derived formulae can be used for the solution of system (22) with the initial conditions of (23).

However, before we proceed further, the following definition is necessary.

Definition 4 We shall call X_o *a consistent initial condition* for (22) at t_o, if there is a differentiable solution to (22) defined on some interval $[t_o, t_o+\gamma]$, $\gamma>0$ such that

$$X(t_o)=\Phi_o(t_o),\ X'(t_o)=\Phi_1(t_o),....,\ X^{(r-1)}(t_o)=\Phi_{r-1}(t_o) \tag{37}$$

see [14].

Consider, now, for instance an electrical circuit which is in operation at time $t<t_o$. Moreover, at the exact time t_o, the system has initial condition

$$X^{(k)}(t_o^-)=\lim_{t\to t_o^-}X^{(k)}(t)\neq\lim_{t\to t_o^+}X^{(k)}(t)=X^{(k)}(t_o^+), \tag{38}$$

for $k=1,2,...,r-1$, which is profoundly non - consistent with the (new) system. This result is due to the impulsive behaviour of system (22) at time t_o, which is translated to an effort to change (almost) instantly, i.e. in zero time, the state of the system in a new initial condition.

From a mathematical point of view, this approach can be modelled efficiently by using the Dirac δ-function and its derivatives. However, in this paper, only the first case is considered and fully discussed.

The results of this section will be applied in order to solve higher order linear systems of type (22). Indeed, the following Theorem divides our initial system (22) into two equivalent, lower dimension differential systems.

Theorem 1 System (22) is divided into two subsystems

$$Y_{p,n}^{(r)}(t)=J_pY_{p,n}(t-\tau) \tag{39}$$

with initial conditions $Y_{p,n}(t_o), Y'_{p,n}(t_o),....,Y_{p,n}^{(r-1)}(t_o)$,

and

$$H_qY_{q,n}^{(r)}(t)=Y_{q,n}(t-\tau) \tag{40}$$

with initial conditions $Y_{q,n}(t_o), Y'_{q,n}(t_o),....,Y_{q,n}^{(r-1)}(t_o)$.

Proof We make the transformation

$$X(t)=QY(t). \tag{41}$$

Then, system (22) is transformed to

$$FQY^{(r)}(t)=GQY(t-\tau). \tag{42}$$

Multiplied by left by the non-singular matrix P, we obtain

$$PFQY^{(r)}(t)=PGQY(t-\tau)\Leftrightarrow$$

$$F_wY^{(r)}(t)=G_wY(t-\tau)\Leftrightarrow$$

$$\begin{bmatrix} I_p & \mathbb{O}_{p,q}\\ \mathbb{O}_{q,p} & H_q\end{bmatrix}\begin{bmatrix} Y_{p,n}^{(r)}(t)\\ Y_{q,n}^{(r)}(t)\end{bmatrix}=\begin{bmatrix} J_p & \mathbb{O}_{p,q}\\ \mathbb{O}_{q,p} & I_q\end{bmatrix}\begin{bmatrix} Y_{p,n}(t-\tau)\\ Y_{q,n}(t-\tau)\end{bmatrix}. \tag{43}$$

Then, eq. (39) and (40) are derived.

Now, the initial conditions are obtained

$$X(t_o)=QY(t_o)\Leftrightarrow Y(t_o)=Q^{-1}X(t_o)=Q^{-1}\Phi_o(t_o),$$

$$X'(t_o)=QY'(t_o)\Leftrightarrow Y'(t_o)=Q^{-1}X'(t_o)=Q^{-1}\Phi_1(t_o)$$

$$,...,$$

$$\begin{gathered} X^{(r)}(t_o)=QY^{(r)}(t_o)\\ \Leftrightarrow Y^{(r)}(t_o)=Q^{-1}X^{(r)}(t_o)=Q^{-1}\Phi_{r-1}(t_o)\end{gathered}. \tag{44}$$

The $Y(t_0)=\begin{bmatrix} Y_{p,n}(t_0)\\ Y_{q,n}(t_0)\end{bmatrix}$.

Thus, the initial conditions for system (33) are given by

$$\begin{aligned} Y_{p,n}(t_o)&=\left[Q^{-1}\Phi_o(t_o)\right]_{p,n},\\ Y'_{p,n}(t_o)&=\left[Q^{-1}\Phi_1(t_o)\right]_{p,n},\\ &....,\\ Y_{p,n}^{(r-1)}(t_o)&=\left[Q^{-1}\Phi_{r-1}(t_o)\right]_{p,n}.\end{aligned} \tag{45}$$

and for system (34) are provided by

$$\begin{aligned} Y_{q,n}(t_o)&=\left[Q^{-1}\Phi_o(t_o)\right]_{q,n},\\ Y'_{q,n}(t_o)&=\left[Q^{-1}\Phi_1(t_o)\right]_{q,n},\\ &....,\\ Y_{q,n}^{(r-1)}(t_o)&=\left[Q^{-1}\Phi_{r-1}(t_o)\right]_{q,n}.\end{aligned} \tag{46}$$

□

5. Conclusions – Further Results

In the present work the motivating reasons as well as the necessary mathematical background have been introduced and presented for the study of higher order linear [(non-) homogenous] descriptor delay differential systems.

This general class of ordinary differential equations can cover both linear first order differential systems and Network Controlled Systems, i.e. system with delays.

Since, our main goal was to motivate our research efforts, we avoided in providing complicated mathematical formulae. In the next lines, further research directions are numerated and briefly presented.

a) We want to investigate the solution properties of different kind of systems (see below) using two well-known and distinguished approaches. The first one is based on matrix pencil theory and the other is based on Drazin inverse theory. The links and comparisons between the results of those methods are expected to be examined. For more information, see section 5, where some preliminary results and relevant mathematical background is provided.

b) Analytically, we are interested in the following systems:

- $FX^{(r)}(t) = GX(t-\tau)$ (see also 5th section),
- $FX^{(r)}(t) = GX(t-\tau) + BU(t)$ (i.e. the non - homogeneous case, where $U(t)$ can be considered as different kind of controllers, feedback, PI, PDI etc.)
- $FX^{(r)}(t) + A_{r-1}X^{(r-1)}(t) + ... + A_1X(t) + A_o = BU(t)$ (where a much more general system is derived)

c) As it has been already mentioned at the end of the 3rd section, it would be very interesting to consider matrices of a special structure, for instance, positive (or negative!), non-negative, symmetric, skew symmetric etc, see for instance [22].

d) Moreover, following recent results presented in [5], [17], [22] and [26], and extending (a), analytical and computational easy formulae for the solutions of the system in (c) are expected to be investigated.

e) From a control point of view, different extensions can be considered. For instance, robustness and different kinds of stability are expected to be further investigated. It is very interested, see also [10], for the descriptor case, where several -mathematically speaking- challenging types of stability have been established. The extension to higher order systems is not an easy task.

f) Moreover, as it is already discussed in definition 4, see 5th section, for descriptor systems, it is important to consider two separate cases when the consistency and non-consistency of the initial conditions can lead us to a distributional expression for the solution. Serious additional effort is needed for the case of non-consistent initial conditions. This case appears when we have rectangular coefficient matrices.

g) Finally, but yet importantly, many numerical issues need to be addressed and simulations and algorithmic schemes should be considered and created. The connection of theoretical results with the mathematical findings is needed and required. For example, it is not clear what type of interactions to the state parameters can appear in NCS's. It would be also very interesting in extending the practical results to different engineering application disciplines, such as the chemical engineering industry, the machining and metallurgical engineering processes, as well as the fast emerging field of remotely operated robots.

For all of these application areas, some preliminary results and thoughts have been proposed and implemented. However, much additional work is needed. We hope that this article provides a further concrete step towards this direction.

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