MODELLING AND SIMULATION OF THE ACTIVE VIBRATION CONTROL BY MEANS OF PI REGULATED ELECTRO-DYNAMIC ACTUATOR AND BOND GRAPH APPROACH

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Abstract

The implementation of active vibration control to the two dof system by means of PI controller is modelled and simulated using bond graphs and conventional multi-physics matrix approach. The active vibration control of the transient and harmonic excitation is considered. The analysis has been performed in time and frequency domain. The results are correlated to the experiments on the real model.

Keywords: active vibration control, electrodynamic exciter, PI control, discrete system

Presenting Author's biography

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1 Introduction

The implementation of the active vibration control to the two dof mechanical system by means of PI controller is modeled and simulated using bond graphs and conventional multi-physics matrix approach. The active vibration control of the transient and harmonic excitation is considered. The analysis has been performed in time and frequency domain. The results are correlated to the experiments on the real model.

The governing equation of the linear mechanical system reads:

$$\mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\zeta}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q} \tag{1}$$

where **M**, ζ and **K** are *mxm* system mass, damping and stiffness matrices, respectively and **q** and **Q** are generalised coordinates and generalised forces. Providing that boundaries, deformation geometry and material do not involve the nonlinearities, the system model will be governed by linear equations i.e. matrices **M**, ζ and **K** do not depend on displacements and their derivatives, [1]. The equation has dimension of force: inertial forces, viscose forces, elastic forces and external forces, respectively. The size of the system matrices in generalized coordinates is m whereas $n=2m$ is the problem size in state space presentation. The generalised displacements are defined as the minimal set that describes position of the structure in unique way. Typically, to solve problem means: find position $\mathbf{q}(t)$ for known forces $\mathbf{Q}(t)$ that satisfies governing equation (1) and initial conditions $\mathbf{q}(0) = \mathbf{q}_0$ and $\dot{\mathbf{q}}(0) = \dot{\mathbf{q}}_0$. The indirect problem, finding forces **Q** for given motion **q** is straightforward.

The control tasks traditionally involve completely new terminology and aspects. The governing equation:

$$\dot{\mathbf{x}} = \mathbf{A}_c\mathbf{x} + \mathbf{B}_c\mathbf{u}(t) \tag{2}$$

where $\mathbf{A}_c$ and $\mathbf{B}_c$ are system matrices, $\mathbf{u}(t)$ is input and $\mathbf{x}(t)$ is vector of state variables, is said to be in state space. For the linear mechanical system (1) corresponding system in state space is linear and time invariant i.e. system matrices $\mathbf{A}_c$ and $\mathbf{B}_c$ are constant and state vector typically has the form $\mathbf{x}^T = \begin{bmatrix}\mathbf{q}^T & \dot{\mathbf{q}}^T\end{bmatrix}$, [2]-[3]:

$$\mathbf{A}_c = \begin{bmatrix} 0 & I \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\boldsymbol{\zeta} \end{bmatrix}.$$

$$\mathbf{B}_c = \begin{Bmatrix} 0 \\ \mathbf{B}_2 \end{Bmatrix}$$

The state variables make minimal set of variables that is supposed to be sufficient to find system state $\mathbf{x}(t)$ at instant t for given state $\mathbf{x}(t_0)$ at instant t_0, so in generic form we have:

$$\mathbf{x}(t) = e^{\mathbf{A}_c(t-t_0)}\mathbf{x}(t_0) + \int_{t_0}^{t} e^{\mathbf{A}_c(t-\tau)}\mathbf{B}_c\mathbf{u}(\tau)\mathrm{d}\tau \tag{3}$$

When designing a new structure, the structural dynamic engineers use (1) more frequently than (2) due to number of reasons: the tradition and the available huge number of efficient numerical methods are some of the most important. However, when we need feedback from the real/realized structure, it comes in the form:

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}(t) \tag{4}$$

Where matrices **C** and **D** represent (hopefully linear) properties of sensors. Having output in the form:

$$\mathbf{y} = \mathbf{C}_a\ddot{\mathbf{q}} + \mathbf{C}_v\dot{\mathbf{q}} + \mathbf{C}_d\mathbf{q}$$

where $\mathbf{C}_a$, $\mathbf{C}_v$, $\mathbf{C}_d$ are output influence matrices, we find:

$$\mathbf{y} = \mathbf{C}_a\left[-\mathbf{M}^{-1}\boldsymbol{\zeta}\dot{\mathbf{q}} - \mathbf{M}^{-1}\mathbf{K}\mathbf{q} + \mathbf{B}_2\mathbf{u}(t)\right] + \mathbf{C}_v\dot{\mathbf{q}} + \mathbf{C}_d\mathbf{q}$$

where $\mathbf{B}_2$ is force influence matrix.
The number of the activities related to the structure development: structure model validation, structure model update, control of smart structures and so on relay on (1), (2) and (4).
Although, the procedure of forming model in state space (2) for given (1) is simple and straightforward it is not unique. Hopefully, the modal characteristics are kept in transformation form (1) to (2) and this is the most often basis for structure monitoring, damage detection and so on, [4].
State space discrete time model presentation, deduced from (2) and (3), reads:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) \end{aligned} \tag{5}$$

where:

$$\mathbf{A} = e^{\mathbf{A}_c\Delta t}$$

$$\mathbf{B} = \int_0^{\Delta t} e^{\mathbf{A}_c\tau}\mathbf{B}_c\mathrm{d}\tau$$

and $k\Delta t = t$.
Here, the basic terminology related mathematical model of the controlled structure is introduced and we proceed to the problem modelling and simulation.

The modelling and simulation of the linear systems are well developed and researched. However, different fields in engineering typically have independent development strategies and paths due to historical reasons. The exchange of ideas in forms of the knowledge transfer has enhanced frequently the domain.

2 The mathematical model of the two bar mechanical system

The two bar mechanical system that will be actively controlled is presented on the Fig. 1. The expected excitation is harmonic, random or impact force and they are applied at the lever 1.

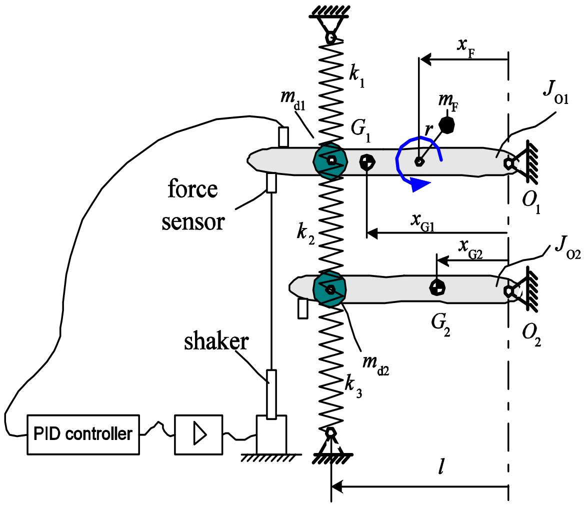


Fig. 1: Mechanical system with two dof, An electro-dynamical exciter performs vibration due to PI controller.

The active vibration controls of the permanent (harmonic and random excitation) and transient responses (impact excitation) are evaluated respectively.

The governing differential equation that corresponds to the actively controlled mechanical system reads:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F}$$

Two beams are represented as lumped masses.

The system properties and values of the corresponding matrices are crucial in reliable modelling and simulation. So, an extensive identification procedure is implemented in order to define system parameters. However, identification procedure itself requires the model of the system and multiple simulations that are required by the optimisation procedure. Typically, the identification is inverse procedure where the minimisation of the objective function leads to the fitting of the real system response.

Although the simple idea, this inverse procedure is numerically challenging due to great number of local minima with close values, even for a small systems. The optimisation procedure is case sensitive and depends also on the fitting targets: time or frequency response. Some possible targets are:

•Real part of FRF $\mathrm{Re}(H_{ij})$

•Imaginary part of FRF $\mathrm{Im}(H_{ij})$

•Module of FRF $Abs(H_{ij})$

•Argument of FRF $Arg(H_{ij})$

•Time response $X(t), \dot{X}(t), \ddot{X}(t)$

•MAC Φ

•The simultaneously weighted combination of the different characterization of the system…

The objective function depends on the system parameters such as mass, stiffness, damping,… (**M**, **K**, **D**, $\mathbf{\Omega}^2$, Φ, ξ,…)

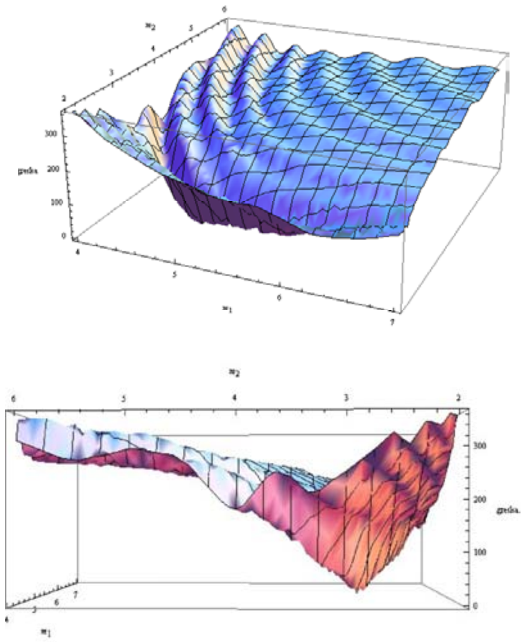


Fig. 2: Objective function example, Error norm as function of m_1 and m_2

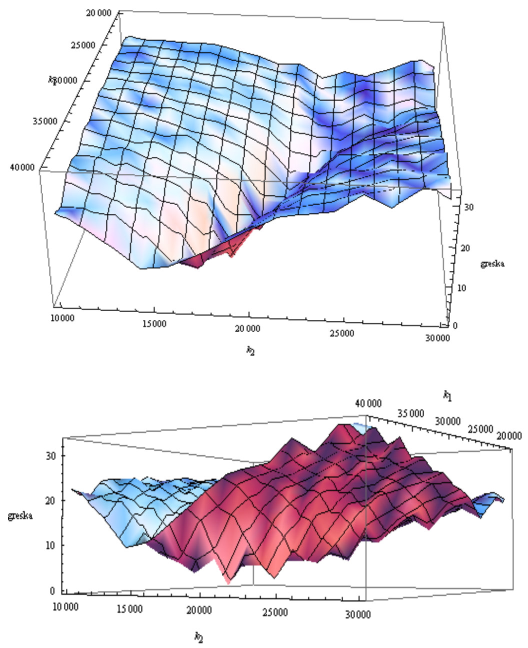


Fig. 3: Objective function example, Error norm as a function of k_1 and k_2

The examples of objective functions are presented in Figs 2 and 3 where the character of these functions can be observed.
Le us stress the fact that we are going to model and simulate the real system and we need the parameters of the system.

The system model can be built on the knowledge of the physical processes of the system. The parameters of the system can be acquired from the known material data. The practical experience in modelling of the real systems has proved that a significant improvement can be achieved in the model update with tuning model with real system responses.

However, to perform the model update we need the model of the system that we are going to simulate. In this way the whole modelling procedure appears to be an iterative algorithm.
Applying identification procedure, the system is identified, and we have mass matrix, [5]:

$$\mathbf{M} = \begin{bmatrix} 4.8402 & 0 \\ 0 & 2.8288 \end{bmatrix} \text{kg}$$

and stiffness matrix:

$$\mathbf{K} = \begin{bmatrix} 35.1318 & -13.5843 \\ -13.5843 & 18.7371 \end{bmatrix} \cdot 10^3 \ \frac{\text{N}}{\text{m}}$$

The type of the system damping is unknown but as far as we are dealing with the real system damping exists. The origins of the friction are the beam supports, spring contacts and internal friction. So, for the real system we are going to model and simulate, viscous – proportional damping is assumed:

$$\mathbf{D} = \alpha \cdot \mathbf{K} + \beta \cdot \mathbf{M}$$

The coefficients of the damping α and β are detected within identification process, we find: $\alpha = 0.05$ and $\beta = 0.0007$

Now we have damping matrix:

$$\mathbf{D} = \begin{bmatrix} 24.8343 & -9.509 \\ -9.509 & 13.2574 \end{bmatrix}$$

The identification process is based on the ERA procedure (Eigensystem Realisation Algorithm). The ERA was used in combination with the observer Kalman filter in order to detect the system modal parameters and verify stiffness, mass and damping.

Transfer function of the controlled system/process is:

$$\mathbf{G}(s) = \frac{1}{\mathbf{M}s^2 + \mathbf{D}s + \mathbf{K}}$$

The block diagram of the controller and controlled system is presented in Fig. 4.

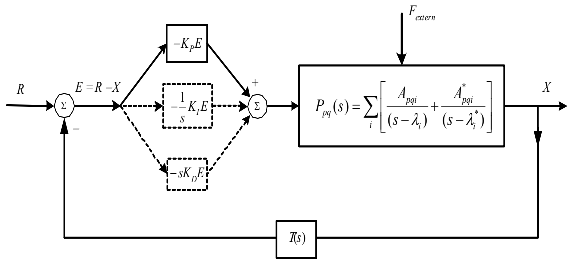
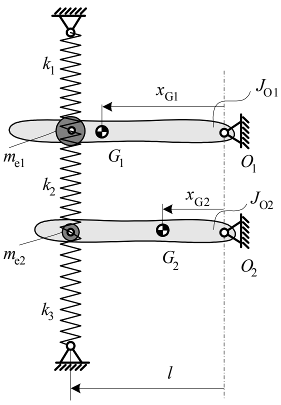
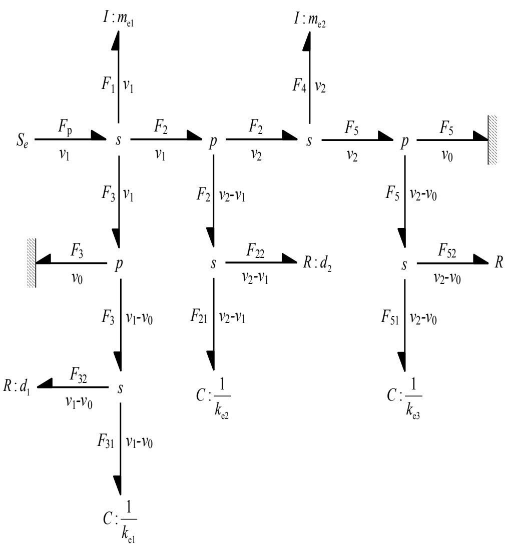


Fig. 4: Schematic presentation of control loop with the process ($P(s)$, mechanical structure with external excitation) and transfer function $T(s)$ for the sensor (transducer)

In Fig. 4 the PID controller is presented. However, only PI control is modelled and simulated in this paper.



a) Mechanical model scheme

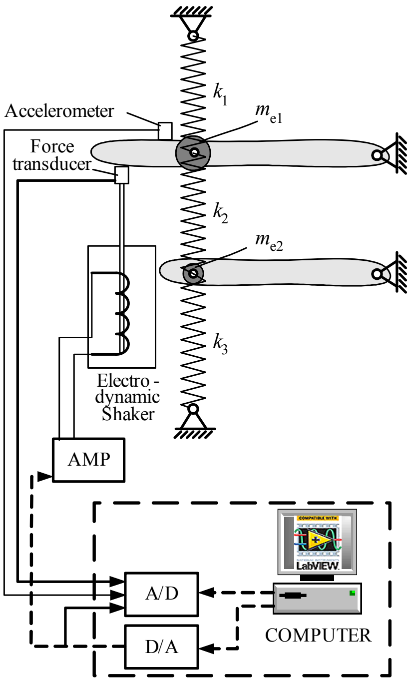


b) Bond graph model

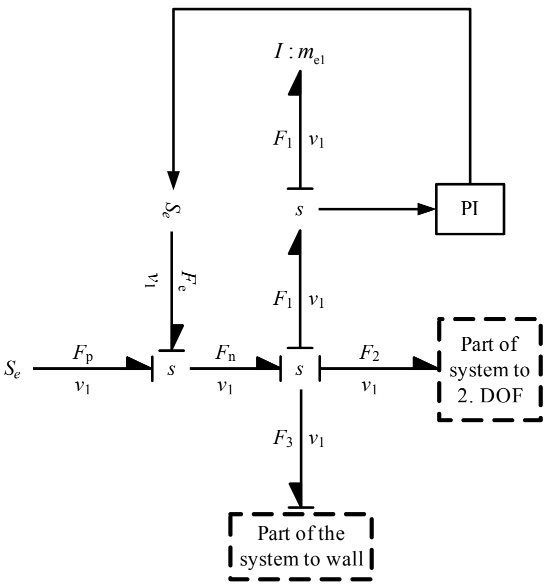
Fig. 5: Mechanical model and corresponding Bond graph

The iterative identification/mode update procedure is applied in this case. The system is modelled based on the physical knowledge of the processes and than iteratively improved until satisfactory data are reached.

The important issue rise: how we can know when the model is appropriate in the case when we do not have built the physical model yet. Here, the experience in building the similar models play very important role.



a) The schematic presentation of the mechanical system digital control



b) The mechanical system and PI control

Fig. 6: The implemented control and Bond graph model

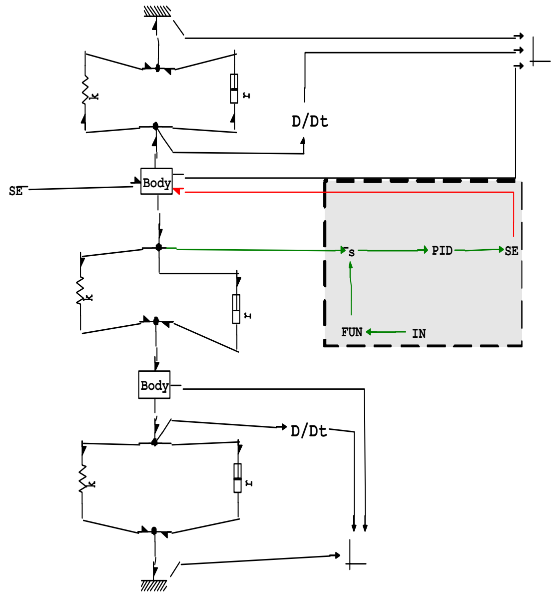


Fig. 7: Bong graph model of 2 dof lever mechanical system in BondSim

The bond graph models are shown in Fig. 5 to 7. The bond graph models are typically transformed to state space model and then solved. The used BondSym applies different methodology: bond graphs are converted in the system of differential and algebraic equations and then solved simultaneously, [6].

3 The simulation and the control

The PI controller is implemented as analogue and digital, see Fig 6 and 8. The controller is tuned using Ziegler-Nichols practical procedure, [7 – 9]. The same coefficients are used for real setup and numerical simulation. These coefficients were starting point for the system optimization.

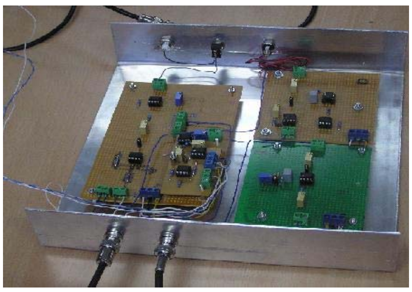


Fig. 8: The analogue implementation of the PI control

The simulation has been performed using BondSim on shown Bond graph model.

The advantage and disadvantage of specific modelling and simulation methods can bee heavily dependent on the software implementation and user previous experience. So, we do not have preference regarding the implementation of the bond graph method over the more conservative ones like matrix approach. However, the bond graphs are the suggestive encapsulation tool and give the strong device feeling especially in graphic presentation.

The optimised parameters of the controller depend on the loading conditions of the mechanical model. The all three loading conditions require dedicated optimisation. Also, multi-objective optimisation can be performed to reach good-weighted optima.

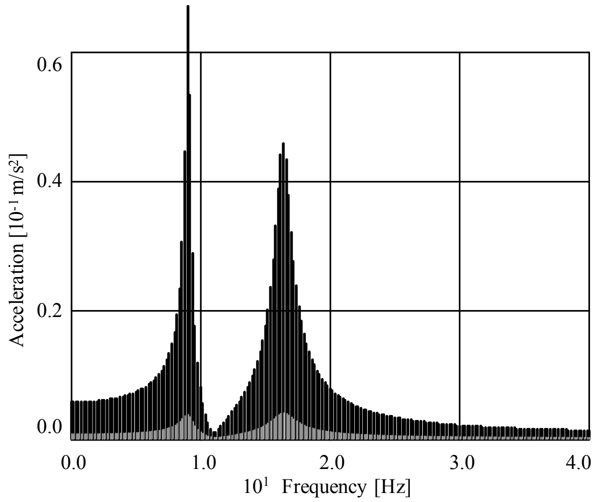


Fig. 9: FRF of bond-graph model using BondSim. Black response is for free system and grey is for controlled.

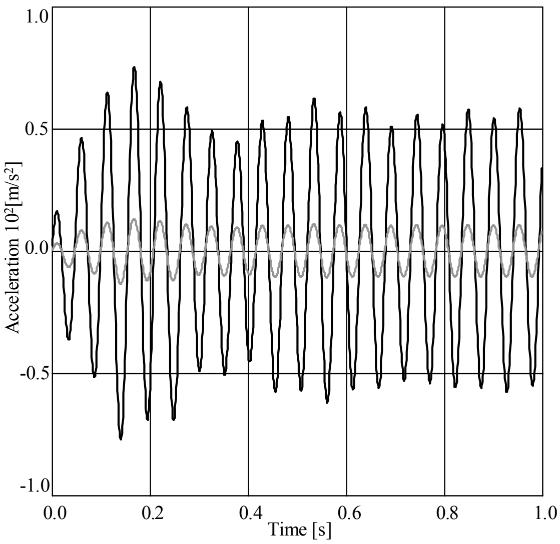


Fig. 10. Simulated results of mass unbalance for frequency of 120 rad/s. Black line stands for free system and gray stands for controlled system.

The response of the open loop system and the optimized PI controlled system are presented in frequency domain, see Fig. 9. The Fig. 10 represents the system response to the harmonic excitation at 120 rad/s for both: the open loop and the PI controlled system.

The strong impact of the active control can be observed. The active control is more efficient in the resonant frequency domain where mechanical systems have lower dynamic stiffness.

4 Measurement on the real model

The measurements have proved simulation results as expected as far as the parameters of the model have been retrieved in identification. The additional assessment of the control efficiency has been performed measuring transient response on the real model.

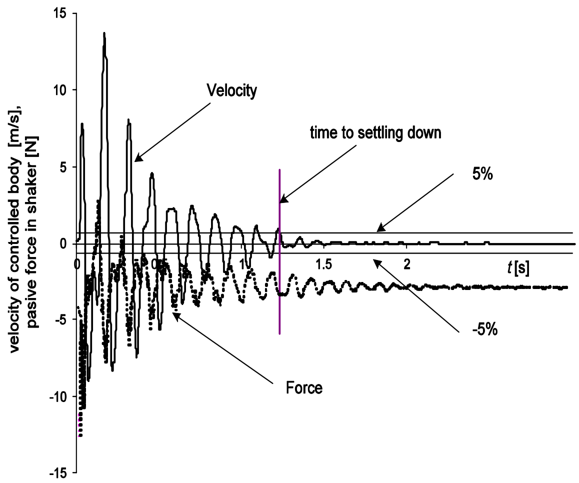


Fig. 11: Settling down due to step excitation (displacement): Measured open loop system; abscise–time in s, ordinate–passive shaker force in N and velocity in ms^{-1}

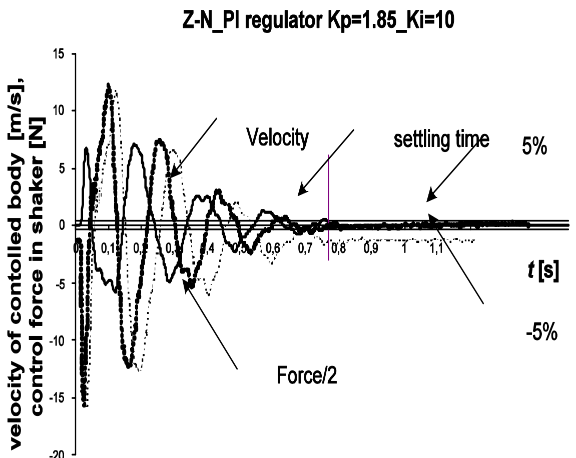


Fig. 12: Settling down due to step excitation (displacement); Measured response of actively controlled system (PID parameters: K_p=1.85, K_i=10); abscise – time in s, ordinate – the half of force in shaker in N (the half of the force fit in the same scale with velocity) and velocity of controlled body in ms^{-1}

The Fig. 11 and 12 represents response of the system to the step excitation. In Fig. 11 we have the open loop response. However, some passive vibration damping in exciter is inevitable, because the actuator is connected to the mechanical system end have damping due to friction and the airflow. The impact of the active control can be observed in Fig. 12, where we see that the settling time as well as peak velocity value is reduced approximately to half of the undamped value.

The identification of the closed loop system has not been performed.

5 Conclusion

The implementation of the active vibration control to the two dof system by means of the PI control are modeled and simulated using bond graphs and conventional multi-physics matrix approach. The identified parameters of the open loop mechanical system are used in simulation of the active PI control of vibration. The active vibration control is implemented to the transient and harmonic excited mechanical system. The analysis of the active vibration control has been performed in time and frequency domain.

The optimised parameters of the controller depend on the loading conditions of the mechanical system. The harmonic excitation and transient step excitation of the mechanical system are considered.

The advantage and disadvantage of the bond graph approach has been considered. At the given level of the application, only the practical and subjective differences are observed in comparison with conservative multi-physics approach.

The identification of the open loop system has been performed, however, the identification of closed loop system is gong to be done in the future research. The evaluation of the active vibration control efficiency is assessed based on displacement and velocity amplitude reduction.

6 References

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