RULE-BASED INTERVAL VALUED FUZZY LOGIC SYSTEM

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Abstract

Today road transport models (microscopic, mesoscopic and especially macroscopic) are not respecting influence of driver mental processes or are strongly simplifying them. Human brain processes uncertain information from imprecise sources on the base of incomplete model of environment, other driver behaviors, skills and features, traffic situation, etc. The big problem of today development is to model human reasoning under conditions of incomplete, imprecise, insufficient or vague information and to bring human mental models and human information processing into our transportation models. It limits today models predictive capabilities. This paper presents novel rule-based description extending well known Takagi-Sugeno-Kang system and implementing Interval Valued Fuzzy Sets. The main difference to standard Takagi-Sugeno-Kang rule system is that left side of the rule does not describe position of one singleton but whole set of parameters determining positions of fuzzy number landmarks. Non-standard interpretation of resulting fuzzy set called granulation replaces usual defuzzyfication to improve understanding of uncertainty of the result. The system is used for human driver mental model development to improve today road transport systems because it is able to describe change of uncertainty. For example, uncertainty of future trajectory of neighborhood car changes with quality of street surface, speed weather conditions, etc.

Keywords: Transport modeling, Mental models, Fuzzy set, Rule, Fuzzy linguistic variable, Granulation.

Presenting Author's biography

Tomas Brandejsky is head of Department of informatics and telecommunications of Faculty of Transportation Science at Czech Technical University in Prague. He received Ing in mechanical engineering and PhD in Cybernetics at CTU. He publish many papers in Soft computing, especially fuzzy sets, genetic algorithms and genetic programming. He also works in area of Artificial Intelligence and Software reliability and safety.

1. Introduction

Today road transport models (microscopic, mesoscopic and especially macroscopic) are not respecting influence of driver mental processes or are strongly simplifying them. Human brain processes uncertain information from imprecise sources on the base of incomplete model of environment, other driver behaviours, skills and features, traffic situation, etc. One (not only) way of studying its behaviors is the application of second order fuzzy sets which are ready to describe both possibilistic and probabilistic uncertainty (see [1]). Because the application of second order fuzzy sets is complicated and unintuitive and because many problems are easier described by set of rules than by set operations, in this paper different approximation will be used. There are two so-called interval valued fuzzy sets theories: First of them are Intuitionistic fuzzy sets [2], which are based on the idea of membership and non-membership functions. The main problem is, that there is not always explained where do these two membership functions come from. Second approach is based on idea of upper and lower estimation of membership function position. These Interval-valued fuzzy sets were introduced independently by [1], Grattan-Guiness [3], Jahn [4], Sambuc [5], in the seventies, in the same year. The work [6] suggests natural way how to explain origin of these two limits on the base of implication operation features. Both approaches are equivalent, as it is presented by a lot of works; see e.g. [7].

2. Novel model outline

The main advantage of interval valued fuzzy sets (IFS) in modeling is their respect to the fact that partially untrue rules tends to raise of second kind of uncertainty – probabilistic (stochastic) uncertainty. Because fuzzy rule sets are used in system modeling as universal approximators, where only typical situations are described explicitly in the form of rules and the rest of features and behaviors is approximated, this feature allows more precise modeling of uncertainty in driver reasoning process. Unfortunately, e.g. in driver reasoning modeling, the used set of rules is incomplete and thus, the precision of interpolation and conclusions based on it is limited and it is need to model this uncertainty. In many situations as human operator mental model description it is also need to use partially untrue rules which tends to uncertainty of conclusions due to features of implication. This situation directly produces probabilistic uncertainty of fuzzy model conclusions.

Many researches incorporate Takagi - Sugeno – Kang (TSK) model, see [8] and [9]. This model is popular in the area of fuzzy modeling and it is based on idea of singleton (double of crisp value and membership value or single element fuzzy set). Precise position of the singleton is determined by linear algebraic function and while standard fuzzy rule has the form (1) the TSK rule has form (2):

IF <fuzzy proposition> THEN <fuzzy proposition> (1)

IF A is a and … and Z is z THEN y=f(A,…,Z) (2)

TSK models were developed for application in such control systems and models, where output variable is defuzzyfied, where no information about uncertainty and its distribution is required. Unfortunately, they are not frequently used in transportation models, where from historical consequences (macroscopic models, data analysis) probabilistic models are preferred. Theory of fuzzy rule based reasoning system based on interval valued fuzzy sets is not developed yet, thus this paper will be presented such novel system. At first, it is need to define n-parameter fuzzy number. This definition will help us to form IFS Rule Based Model. Then we will briefly discuss interval valued fuzzy sets. On the base of this discussion, IFS Rule Based Model will be formed.

2.1. N-parameter fuzzy number

Each fuzzy number is n-ary projection from universe space U_n into real number interval <0,1>. It is possible to describe it as function of n parameters. Standard fuzzy numbers which do not change its position are functions of 0 parameters non-looking that they are singletons, triangular, rectangular or any else fuzzy numbers. In standard TSK models, 1-parametric fuzzy numbers in form of position changing singletons are used. Presented model differs in use of n-parametric fuzzy numbers changing not only position, but the shape too. Fig. 1 and 2 bring examples of 1 and 2 parameters fuzzy numbers for arguments taking magnitudes 0, 0.5 and 1.

2.2. Interval Valued Fuzzy Sets

Presence of more than fifty logical system in Fuzzy Logic, the most known are listed in Table 1, underlines hidden ambiguity in fuzzy set definition, which tends not only into definition of so-called T-norms (generalized intersection) and T-conorms (generalized conjunction) but rises probabilistic uncertainty of some results. As solution, it is possible to accept above mentioned interval valued fuzzy sets, where implication on the base of partially untrue rule generates whole set of solutions.

IVFS enables to model situations, where more solutions of one given problem are possible. It is possible to apply IVFS to model uncertainty concluding from operator observation, especially when this observation is incomplete and when rules are partially incorrect. Such situations are frequent especially in road transport, where cognitive ability of the driver are limited and in some situations insufficient.

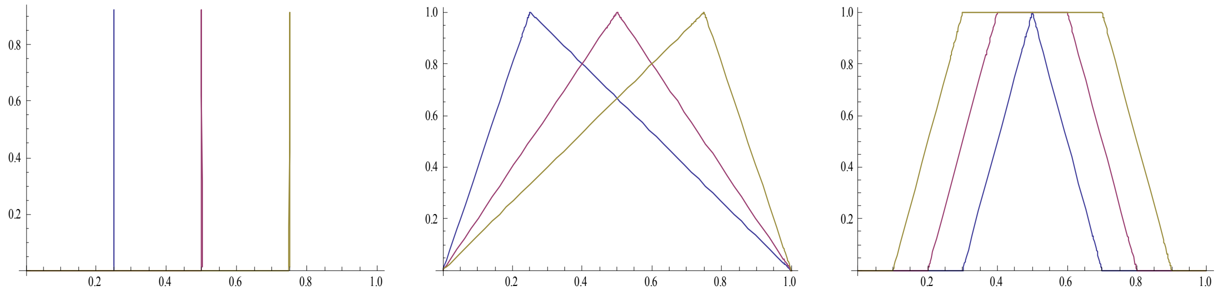

Fig. 1: An example of 1-parameter fuzzy numbers (position changing singleton in style of TSK model, shape and gravity centre changing triangular fuzzy number and fuzzyness changing trapezoidal fuzzy number).

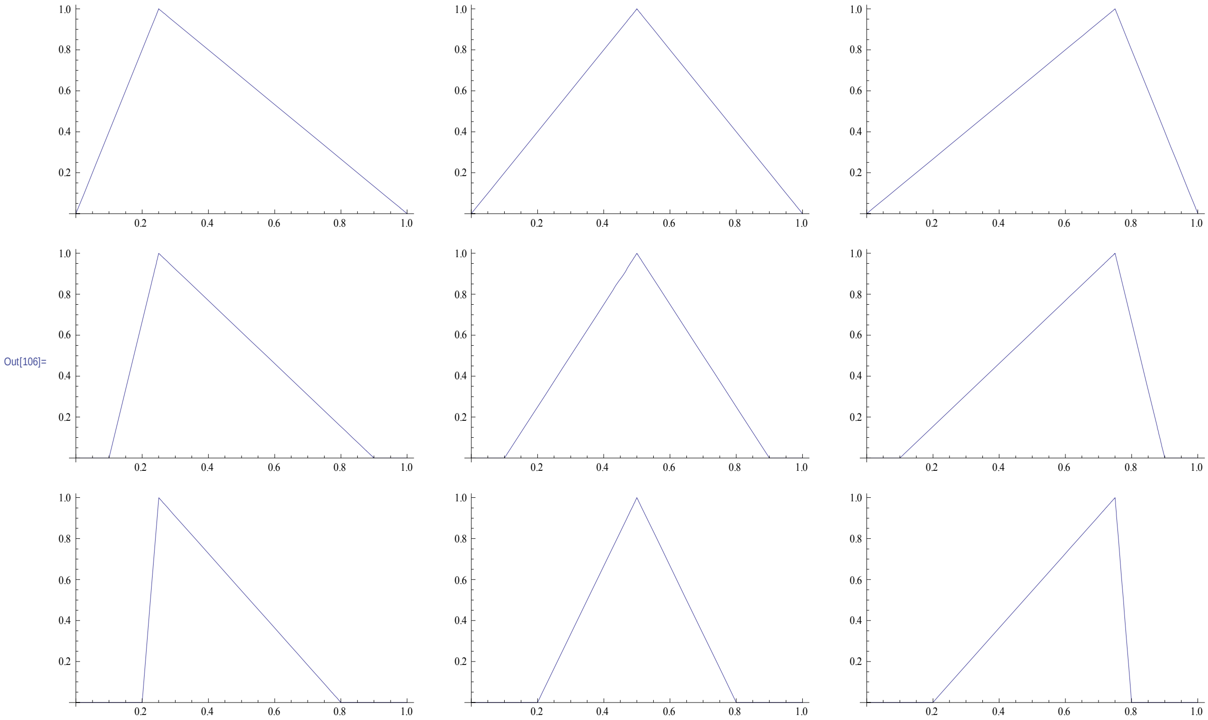

Fig. 2: An example of 2-parameter fuzzy number (triangular fuzzy number moving peak position and fuzzyness).

Tab. 1: Examples of fuzzy implications in most frequently used fuzzy logical systems

Larsen	$x \to y = xy$
Łukasiewicz	$x \to y = \min\{1, 1 - x + y\}$
Mamdani	$x \to y = \min\{x, y\}$
Standard Strict	$x \to y =$ 1 if $x \le y$ 0 otherwise
Godel	$x \to y =$ 1 if $x \le y$ y otherwise
Gaines	$x \to y =$ 1 if $x \le y$ y/x otherwise
Kleene-Dienes	$x \to y = \max\{1 - x, y\}$
Kleene-Dienes-Łuk.	$x \to y = 1 - x + xy$

2.3. IVFS Rule Based Model

TSK models were developed for application in such control systems and models, where output variable is defuzzyfied, where is required no information about uncertainty and its distribution. Theory of fuzzy rules based on interval valued fuzzy sets with fuzzy number left side is not developed yet, thus this paper will be based on the following propositions:

Singleton is a special case of fuzzy number. In presented novel IVFS Rule Based Model (IVFS-RBM), unlimited shape fuzzy numbers described by list of landmarks will be used. Because position of each landmark is described by particular function, presented rule system gives chance to control and change shape of fuzzy number in relation to fuzzy rule input attributes.

List of landmarks is computed on the base of corresponding list of functions and TSK system (2) is a special case of single element function and landmark lists and thus a special case of presented rule system.

Fuzzy Numbers of n-th order are in the IFS-RBM system described as (3), where symbols a_i are representing landmarks of fuzzy number (e.g. in the case of triangular fuzzy number they can represent leftmost, peak and rightmost values) and μ is the membership maxima magnitude, see Fig. 1 and 2 too.

$$\text{TFN}_n\text{: } (a_o, \ldots, a_n, \mu) \tag{3}$$

Definition 1: IVFS-RBM uses rules in the form represented as (4),

$$\text{IF} \bigcup_i (x_i = a_i) \overset{\mu_r}{THEN}$$
$$(f_0(x_0, \ldots, x_n), \ldots, f_m(x_0, \ldots, x_n)) \tag{4}$$

Where

x_i means crisp input variable

a_i represents linguistic value

f_i is output crisp function denoting position of i-th landmark of proper output fuzzy number TFN.

μ_r measure of rule truth/validity

3. Granulation

Within the presented IVFS-RBM system, granulation is used on the place of traditional defuzzyfication. Defuzzyfication transforms uncertain data produced by rule based model to base type, typically real numbers. There are known many forms of defuzzyfication; probably the most frequent in T-S-K models is weighted average, see (5)

$$r = \frac{\sum_i x_i \mu_i}{\sum_i \mu_i}, \tag{5}$$

where

r is defuzzyfied output real magnitude

x_i represents significant magnitude of i-th linguistic value, e.g. centre of gravity

μ_i represents membership of i-th linguistic value

The main aim of IVFS-RBM is to achieve uncertainty description (information about uncertainty amount and distribution) in decision process. Thus, it gives no advance to use standard defuzzyfication in presented system due to big lose of information about uncertainty distribution and character in this output normalization. In presented system is used granulation of the information. It is transformation of the result, which maps result in form of fuzzy number into standard representation via predefined fuzzy linguistic variable. This transformation enables use of results in further reasoning, e.g. in expert systems, mental models, or even reactive systems in robotic sense as it was firstly published by Brooks [8] (robotic control schemes are usable in drivers models in advanced microscopic simulators).

Fuzzy linguistic value is described with respect to its original Zadeh's description [1] by definition 2:

Definition 2. A *linguistic variable* V is a quintuple of the form

V = (N,G,T,X,S),

where *N,T, X,* G, and *S* are defined as follows:

1. *N* is the name of the linguistic variable V;
2. *G* is a grammar;
3. *T* is the so-called *term set,* i. e. the set linguistic expressions resulting from *G*\
4. *X* is the universe of discourse;
5. *S* is a *T —> F(X)* mapping which defines the semantics of each linguistic expression in *T.*

There are many ways how to define granulation operation, because representation by fuzzy number contains more information than representation by linguistic variable. E.g., it is possible to apply intersection operation, see (6) and following definition 3:

Definition 3: Granulation operation transforms fuzzy number FN described by membership function $\mu_{FN}(x)$ into fuzzy linguistic variable FLV, see previous definition 2. Intersection based granulation calculates intersection of $\mu_{FN}(x)$ and membership functions ${}_{FLV}\mu_k(x)$ of each linguistic value k of the variable FLV:

$$\forall k : {}_{FLV}\mu_k = \max_x \left({}_{FLV}\mu_k(x) \cap \mu_{FN}(x) \right), \tag{6}$$

where ${}_{FLV}\mu_k$ is membership to linguistic value k of linguistic variable FLV. Such definition describes the rise of ambiguity which cannot be solved within first order fuzzy sets and requires IVFS description. E.g., fig. 3 shows two different situations for which granulation gives equal results, approximately μ_k=0.85 for both linguistic values.

3.1. IVFS extension of granulation

Relation (6) represents pessimistic or lowest possible magnitude of memberships to linguistic values. Maximal possible membership might be estimated as maximum of fuzzy number membership in interval of values x given by linguistic value support, see (7).

$${}_{FLV}\mu_k = \left\{ \max_x \begin{pmatrix} {}_{FLV}\mu_k(x) \cap \\ \mu_{FN}(x), \max_x(\mu_{FN}(x)) \end{pmatrix} \right\} \tag{7}$$

3.2. Granulation features

Standard granulation as it is described by (6) is not unambiguous. Relation (7) is not explicit too, because it maps this uncertainty only.

It the moment when memberships to linguistic values are known, it is impossible to distinguish which fuzzy number was granulated, as it is presented by fig. 3. IVFS extension of granulation tries to cover all possible fuzzy numbers.

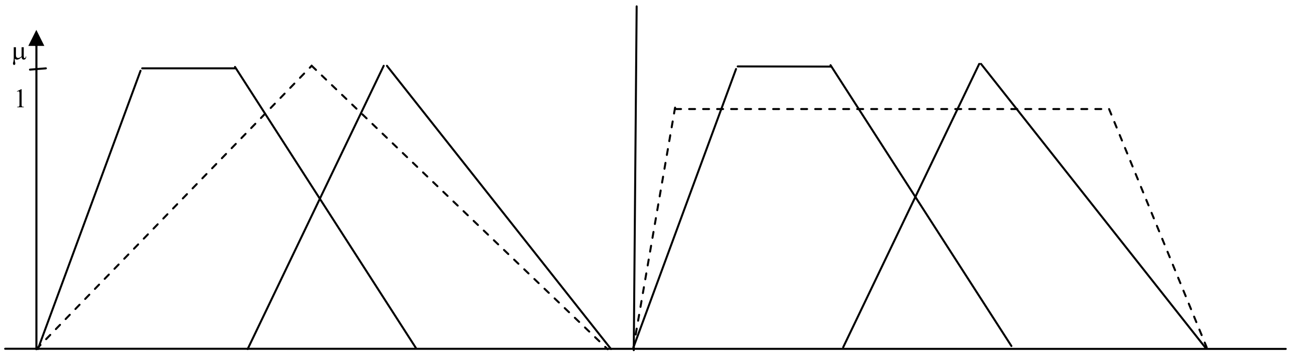

Fig. 3: Ambiguity in granulation solved within first order fuzzy sets.

4. Driver reasoning model examples

Driver reasoning modeling, it means formalization of driver mental models, represents complicated and complex task, which solution might significantly improve today knowledge and understanding of transportation processes, reliability and precision of our models, especially microscopic ones, and give them capabilities to predict behaviors and influence of future changes of transportation systems. Today microscopic simulations are rather ready to animate previously measured transportation situation or to model influence of small changes of transportation systems.

Improvement of today microscopic models requires formalizing of human cognitive and decisioning making processes. Unfortunately, human cognitive capabilities are strongly limited both by nature (human body, nervous system, health, tiredness, and training) and by car design (dead zones, requirement to solve more tasks in parallel). Today traffic also asks to divide driver attention between many neighborhood cars, traffic signs and other information sources. On the opposite side, human brain is capable to keep in short term memory only small number of ideas, objects and plans typically estimated as well known number 7 ± 2 .

Today attempt in this area was limited by our models and related knowledge representation. Big problem brings uncertainty of measured data given by limited precision of our cognitive processes, incompleteness of data caused also by limitation of human brain processing capacity [9], ignoring of some facts due to attention focusing etc. This uncertainty is both possibilistic and probabilistic, thus the representation capable to represent both of them is required.

Second significant problem is given by parallelism of input stimuli and concluding requirement of assigning priority to each task, modeling process of task switching and canceling, influencing of one task by another, especially through emotional reasoning.

Another problem is given by two faces of human reasoning. Solving well known problems we use simple conditional reflexes, as it is described in the work [8]. In the case of more complex problems, complex reasoning process is activated. This process consists of many kinds of reasoning as logical reasoning, abstraction and decomposition, analogical and metaphorical reasoning, paradoxical reasoning, etc, see [10].

Many of these reasoning schemes are not straightforward and tend to complicated reasoning process consisting of many level nested loops and optimization processes. The most significant conclusion to driver reasoning process modeling is that driver are not ready to find optimal solution, because car driving is real time process and time to decision is limited. Frequently, reasoning process stops before finding of the optimal solution and the solution is replaced by suboptimal one discovered within constrained time.

Presented interval valued fuzzy set rule based model is based on first order logic and thus it is applicable in above mentioned kinds of reasoning. As an example it will be applied to part of driver reasoning model – model of car speed change in the case of oncoming car meeting.

4.1. Oncoming car meeting driver reasoning model

In the case of oncoming car meeting drivers reason if the correction of the speed is need or not. Requirement of the speed correction (speed decreasing) depends on feeling of threaten (on probability and possibility of collision risk).

Driver analyses in his decision process such features of the situation as road surface quality q, speed of own car s_t, speed of oncoming car s_n (and thus relative absolute speed s_r of the cars), width of free corridor w for own car, estimated quality of ongoing car diving (degree of motion control) e_q, distance (or more precisely time t_d) to meeting point, etc. Each of these input variables is in presented model represented as fuzzy linguistic variable, see Fig. 4-10:

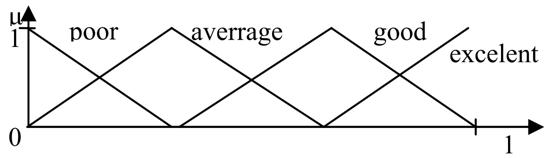

Fig. 4: Road surface quality q.

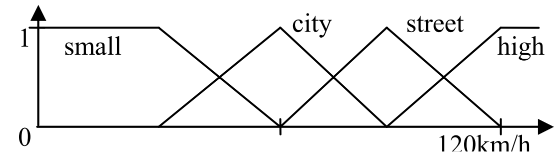

Fig. 5: Speed of own car s_t.

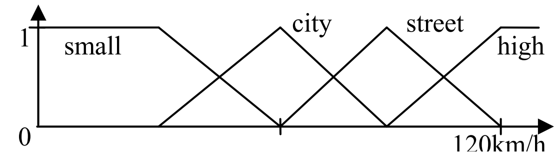

Fig. 6: Speed of oncoming car s_n.

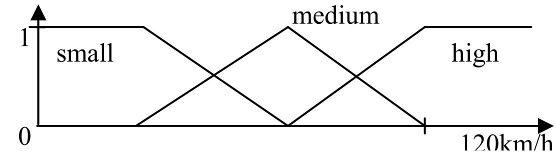

Fig. 7: Relative speed s_r of the cars.

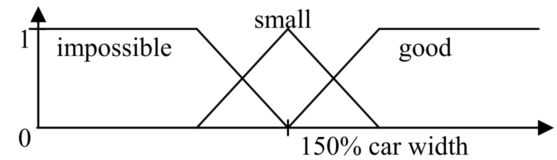

Fig. 8: Width of free corridor for own car w.

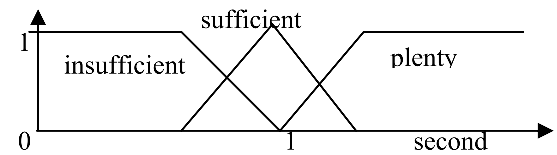

Fig. 9: Time t_d to meeting point.

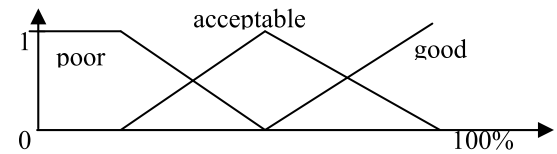

Fig. 10: Estimated quality of ongoing car diving e_q.

State and output variables of the model are sufficiency of free space s_f and risk of negative change r. They are outlined by Fig. 11, 12 and 13 :

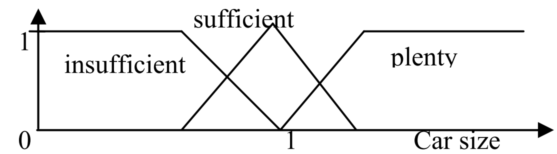

Fig. 11: Sufficiency of free space s_f.

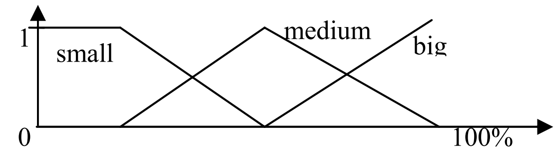

Fig. 12: Risk of negative change r.

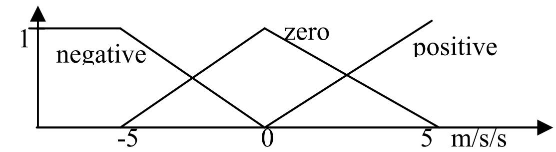

Fig. 13: Speed correction Δv.

Set of IVFS-RBM rules describes relationships between variables and consist of following rules (8-10):
The speed correction is function of sufficiency of free space s_f and risk of negative change r. It implies 9 rules (2 variables of 3 linguistic values each), see (8) for right hand functions of rules:

Suff.\risk	small	medium	Risk
Insufficient	(-max,-max,-max)	(-max,-max,-max)	(-max,-max,-max)
Sufficient	0	(-risk,-risk/2,0)	-risk
plenty	(0,(max-risk)/2,max)	(-risk,0,max)	(-risk,0,0)

(8)

The risk of negative change r is function of estimated quality of ongoing car diving e_q, relative speed s_r of the cars, speed of oncoming car s_n and road surface quality q. Fortunately, the relation was identified to be linear and thus original set of 256 rules might be reduced into single rule (9):

IF e_q AND s_r AND s_n AND q THEN

$$r = \frac{k_1 * (s_r - s_n)}{(k_2 + q + e_q)} \qquad (9)$$

Parameters k_1 and k_2 are chosen to fit r in interval <0,1>.

The sufficiency of free space s_f is a function of width of free corridor for own car w, speed of own car s_t and road surface quality q. Combination of linguistic values of these variables implies 48 rules but it is possible to reason q and s_t as linear numeric parameters and to reduce rule set into three rules, see (10):

s_f	insufficient	sufficient	plenty
	W=impossible	W=small/s_t/q	W=good/s_t

(10)

Because all arguments are fuzzy numbers, also the result will be fuzzy number.

4.2. Experiment

We will reason following model situation described by following magnitudes of model variables:
Q = average
S_f= 90 km/h
S_n= street
S_r=high
W=small
T_d=sufficient
E_q=acceptable
On their base it is possible to compute resulting magnitudes, sufficiency of free space as the first:

$$w = \left\{ \frac{0}{impossible}, \frac{1}{small}, \frac{0}{good} \right\} \qquad (11)$$

Thus
Sf={0/impossible,(0,1,2):μ=1/sufficient,(1/90,2/90,∞)/ plenty} (12)

The risk of negative change r is computed on the base of estimated parameters $k_1 = 1/120$ and $k_2 = 1$. Applying (9) we obtain fuzzy number whose defuzzyfication into linguistic variable r produces result (13):

$$r = \left\{ \frac{0}{small}, \frac{0.27}{medium}, \frac{0.81}{big} \right\} \qquad (13)$$

On the end, speed correction Δv is determined on the base of rule set (8). This rule set produces 9 triangular fuzzy numbers but membership of many of them is equal to zero, see (14):

Suff\risk	Small, μ=0	Medium, μ=0.27	Big, μ=0.81
Insuff., μ=0			
Suff., μ=1		(-1,-0.5,0), μ=0.27	(-1,-1,-1), μ=0.81
Plenty, μ=0			

The interpretation of the result depends on psychological profile of the driver. For example, rational driver selects first maxima of resulting unified fuzzy number, pessimistic driver takes most constraining solution, optimistic driver chooses the less constraining version, etc.

5. Conclusion

Presented paper describes novel approach to interval valued fuzzy sets modeling based on extension of Takagi-Sugeno-Kang model and implementation of novel granulation operator. On the base of this model, the driver reasoning model application is presented. This model is of course limited and it will be extended, precisiated and verified in following research, but the first results are in relation to measured data and the model will be included into experimental microscopic road traffic simulator.
It is also significant that model enables mixing of numerical and linguistic magnitudes in coherent system.

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