ADEQUATE MATHEMATICAL DESCRIPTION OF DYNAMIC SYSTEM: STATEMENT PROBLEM, SYNTHESIS METHODS

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Abstract

The main problem of mathematical simulation is the construction (synthesis) of mathematical model (MM) of motion of real dynamic system which in aggregate with model of external load (MEL) give the adequate to experimental observations the results of mathematical simulation. It was shown that the criterions of choice of good MM of dynamic system separately from choice of right MEL do not exist. Two basic approaches to this problem are selected. Within the framework of one of these approaches some algorithms are offered which allows receive adequate results of mathematical simulation. The different variants of choice model which are depending from final goals of mathematical simulation (simulation or modeling of given motion of system, different estimation of responses of dynamic system, simulation of best forecast of system motion, the most stable model to small change of initial data, unitary model) are considered. These problems are incorrect problems by their nature and so for their solution are being used the regularization methods. For increase the exactness of approximate solution the method of choice of special mathematical models was suggested. The test calculation was executed.

Keywords: Mathematical simulation, Adequacy, Inverse Problem, Regularization.

Presenting Author's biography

Yuri Menshikov. He received the degrees of Ph.D. in Applied Mathematics at Kharkov Polytechnical University, Ukraine on 1979. He is working in Dnepropetrovsk University (Mechanics and Mathematics Faculty) from 1970. Dr. Yuri L. Menshikov is reviewer of conference Mathematical Modeling and he is member of Germany Society for Industrial and Applied Mathematics (GAMM). Research interests of Yu.L.Menshikov include system control, differential equations, variation methods, inverse problems. He is an author and coauthor of two monographies and more than 270 scientific papers in international journals and conference proceedings.

1 Introduction

The problem of synthesis of adequate mathematical description of dynamic system was considered by many authors during the last decades. The contemporary situation in this field has been characterized by R. Shannon, who admits, that despite the extensive literature on a substantiation and study of accuracy of modeling, these questions still remain almost so difficult as well as at the beginning of their development [1]. In the synthesis of adequate models of real systems the decisive question is the conformity (in some reasonable sense) of model output to an expected output of real system. In other words, the author specifies importance of a question of adequacy of the mathematical description to a real process.

We will consider a typical situation which arises during the analysis of new processes or phenomena. It is supposed that real physical process is observed and the records of experimental measurements of some characteristics of this process are given. It is necessary to develop adequate mathematical model of this process for further use.

The situation may be explained with the use of a simple example, presented in [2], where the modeling of inclined mechanism is submitted. The state equations of the tilted mechanism was chosen in the form

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = k_1 x_1 + k_2 x_2 + E(t), \end{cases} \tag{1}$$

where k_1, k_2 are coefficients and $E(t)$ is the step-function.

Then the parameters of mathematical model (1) of system are selected in order to provide conditions, when results of simulation coincide with experiment. In other words adequate mathematical model is being chosen. The analogical investigations was performed in [3,4,5,6].

At first in this paper the full process of synthesis of mathematical description (equations of motion + models of external loads) of real dynamic system and theirs calculations under some conditions will be called as mathematical simulation.

We shall limited to consideration only by dynamic systems (processes) which are being described by the ordinary differential equations.

2 Statement of a problem

By the mathematical description of dynamic systems we understand the differential equations of motion which establish connection between the variables of state $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_{n_1})^T$ of dynamic system (responses) and external load $\tilde{z}$ (input), $(\cdot)^T$ – a mark of transposition. For example, this connection for case of linear dynamic system has a form [7]:

$$\dot{\tilde{x}} = C_1 \tilde{x} + D_1 \tilde{z}, \tag{2}$$

where C_1, D_1 – matrixes with constant coefficients.

Now it is possible to give the definition of adequacy of mathematical description for dynamic systems of type (2) more exactly.

Definition of Adequacy: mathematical description of real process will be considered as adequate with respect to selected variables of state of process model, if under proper limitation on external loads and on the value of variables of state real process with the same additional conditions (initial and boundary) will coincide with experimental measurements of corresponding physical characteristics of real process on selected criteria in given metrics with the accuracy of measurement and exactness of coefficients of mathematical model.

Let's assume that external load $\tilde{z}$ and part of state variables $\tilde{x}_{r_1+1}, \ldots, \tilde{x}_{n_1}$, $r_1 + 1 < n_1$ are unknown. Other part of state variables in the equation (2) is measured by an experimental way. It is assumed that the equation of observation has a form:

$$\tilde{y} = F_1 \tilde{x},$$

where $\tilde{y} = (\tilde{y}_1, \tilde{y}_2, \ldots \tilde{y}_{l_1})^T$, $F_1 = \{f_{ik}\}_{i=1,k=1}^{i=l_1,k=n_1}$ – matrix with constant coefficients of size $n_1 \times l_1$, thus $f_{ik} = 0$, for $k > r_1$. Let's consider in addition for simplicity that the matrix F_1 is diagonal $f_{kk} \neq 0$, for $k \leq r_1$, $f_{ik} = 0$, for $k \neq i$. If the part of external loads are known this case can be reduced to examined if using of linearity of dynamic system.

Let's consider known state variable $x_j(t)$ as two known internal loads $d_j \tilde{x}_j(t)$ and $[-d_j \tilde{x}_j(t)]$, $1 \leq j \leq r_1$, d_j - constant. Such interpretation of state variable allows to simplify initial system. Let's name such transformation by "j-section" of initial system (2).

In some cases after a lines of "j-sections" the initial system (2) will be transformed to some subsystem at which is known one state variable, for example, $\hat{x}_1(t) = \hat{x}(t)$ and all external loads $\hat{z}_k(t), k = 2, \ldots, m_2$, except $\hat{z}_1(t)$, for example, are known. This case is being reduced simply to a case when one external load $\hat{z}_1(t) = z(t)$ is not known only if using of linearity of a subsystem. Thus the received subsystem has a form:

$$\dot{\hat{x}} = C \hat{x} + D z, \tag{3}$$

where $\hat{x} = (\hat{x}, \dot{\hat{x}}, \ddot{\hat{x}}, ..\hat{x}^{(n-1)})^T$, $z = (z, \dot{z}, \ddot{z}, ..z^{(m-1)})^T$,

C, D – matrises with constant coefficients.

With using of impulse transitive function it is possible to write down equality:

$$A_p z = \int_0^t K(t-\tau) z(\tau) d\tau = u(t) = B_1 \hat{x}, \ \hat{x} \in X \ , \quad (4)$$

where $K(t-\tau)$ – known kern, A_p is the operator of the certain structure, $A_p : Z \to U$; $B_1 : X \to U$.

If we return to old state variables the equation (4) will be transformed to a kind

$$A_p z = B_p x_\delta \ , \quad (5)$$

where $B_p x_\delta = B_1 \hat{x} - B_{1,p} x_\delta$, $x_\delta = (\tilde{x}_1, ..., \tilde{x}_{r_1})^T$, $B_{1,p}, B_p$ – operators translating elements $x_\delta \in X$ into U.

At research of concrete dynamic systems the structure of the mathematical description, as a rule, is fixed. However parameters of mathematical model it is necessary to believe given approximately. This error can be appreciated from above and, as a rule, she does not surpass 10 % [2].

At performance of concrete accounts it is necessary to take into account that the operators A_p, B_p, depend continually on a vector of parameters p of mathematical model of motion of dynamic system which are determined approximately with some error, $p \in D$, D is some closed region in R^n. Thus we shall believe that for the normalized spaces Z, X, U the inequalities are carried out:

$$\left\| A_p - A_T \right\|_{Z \to U} \le h \, , \left\| B_p - B_T \right\|_{X \to U} \le d \ , \quad (6)$$

where A_T, B_T – exact operators in the equation (5), h, d – known values.
If with the help of lines of "j-sections" it fails to allocate a subsystem (3) with one external load then the given reasoning lose the sense. If the initial dynamic system (2) has some unknown external loads and for each of them it is possible to receive subsystems such as (3) with one unknown external load, the above mentioned reasoning are valid, however further algorithms of construction of the adequate mathematical description essentially become more complicated.
For successful application of methods of mathematical simulation at research of dynamic systems it is necessary to execute synthesis of the mathematical description of real process which allows to receive results of mathematical simulation appropriate to experimental data [8]. Such result can be achieved by synthesis of "correct" mathematical model of motion of dynamic system and choice of "good" model of external load on this system, if system is open.
The equation (5) we shall consider as basic. Let's assume that the initial data $x_\delta = (\tilde{x}_1, ..., \tilde{x}_{r_1})^T$ are received by an experimental way with some known a priori by an error:

$$\rho_X (x_T, x_\delta) \le \delta \ , \quad (7)$$

where x_T – exact initial data, $\rho_X(\cdot, \cdot)$ there is a distance between elements of functional space X, δ – const, $\delta > 0$, δ - inaccuracy of experiment.
The check of adequacy of mathematical model of dynamic system and models of external loads in this case is reduced to control of performance of an inequality

$$\rho_U (A_p z, B_p x_\delta) \le \varepsilon \ , \quad (8)$$

where $\rho_U(\cdot, \cdot)$ there is a distance between elements of functional space U, ε – const, $\varepsilon > 0$, ε – required accuracy of concurrence with experiment. If the functional spaces are normalized then the inequality (8) can has a form

$$\left\| A_p z - B_p x_\delta \right\|_U \le \varepsilon \ . \quad (9)$$

It is natural that ε there can not be less then size δ.
Characteristic feature for examined problems is that the operator A_p is compact operator [9].
It is obvious that in the case of performance of inequality (8) operators A_p and function z are connected. It is easy to show that infinite set of various among themselves functions z which satisfy the inequality (8) exist at the fixed operator A_p in (8) [8,9,11]. And, on the contrary, at the fixed function z there are infinite many various operators A_p for which an inequality (8) is valid [8,10,11]. Thus, there are no opportunities of a choice of good mathematical model of dynamic system (or process) separately from a choice of correct MEL.
So the synthesis of adequate mathematical description is reduced to solution of some integral equations type (5) under conditions (6) - (9).

3 Synthesis methods

Two basic approaches exist to problem of synthesis of couple MM and model of EL [8,12]:

1) MM is given a priori with inexact parameters and then MEL is determining for which the inequality (6) is valid [8];
2) Some MEL is given a priori and then is choosing MM for which the inequality (7) is satisfy [12,13].

Now we will consider the synthesis of adequate mathematical description in the frame of first approach analysing the process with the concentrated parameters, for which the motion is described by ordinary differential equations of n-order (2).

We assume that some functions of state $x_1(t), x_2(t), \ldots x_r(t),\ r \le n$ in system (2) are obtained from experiment and presented by graphs. Besides, we suppose that some of functions of external loads, for example, $z_1(t), z_2(t), \ldots z_l(t),\ l \le m$ are unknown. According to first approach, it is necessary to develop the construction of such model of external loads components which is characterized by the functions of state $x_1(t), x_2(t), \ldots x_r(t)$ of mathematical model (2), and will coincide with experimental measurements $\tilde{x}_1(t), \tilde{x}_2(t), \ldots \tilde{x}_r(t)$ with inaccuracy of initial data. Such mathematical model of process behavior together with obtained model of external load can be considered as *adequate mathematical description* of dynamic system.

Such method of obtaining of mathematical models of external loads (functions $z_1(t), z_2(t), \ldots z_l(t),\ l \le m$) is determined in literature as a method of identification [14,15]. By the way, physical reasons of occurrence of such external loads are not being taken into account. They are only functions which in combination with mathematical model (2) provide results of modelling, which coincide with experiment with the given accuracy.

It can be shown that in the most practical problems the operator A_p is completely continuous [9].

Thus, a necessary condition for obtaining the equation (5) concerning required external load z_i, is the possibility through a number of "sections" to obtain a subsystem of initial system with one unknown external load z_i and one known state variable $x_j(t),\ 1 \le j \le n$. It is easy to demonstrate an example of system such as (2), in which such opportunity is absent.

Let's denote by $Q_{\delta,p}$ the set of the possible solutions of an inverse problem of identification of model of external load (5) with the fixed operators A_p, B_p:

$$Q_{\delta,p} = \{z : \| A_p z - B_p x_\delta \|_U \le \delta \|B_p\| = \delta_0\}.$$

We assume that for any p and any δ the set $Q_{\delta p}$ contains at least one function which is bounded in Z.

Any function z from set $Q_{\delta,p}$ may be considered as "good" model of external load as far as the function $A_p z$ coincides with $B_p x_\delta$ with accuracy of measurement.

Thus, the operators A_p, B_p and any function from set $Q_{\delta p}$ give a pair which will provide adequacy of results of mathematical simulation with accuracy δ_0.

We shall name the process of determination of $z \in Q_{\delta p}$ as synthesis of *model of external loading by a method of identification* [5,10,11].

However set of the possible solutions $Q_{\delta,p}$ at any δ has a number of specific features (it is actually incorrect problem) [9]. First, and main of them is that this set is not bounded at any δ [9,16].

Let's consider this feature more in detail due to the fact that it leads to a number of unexpected and unusual consequences.

The set $Q_{\delta p}$ contains infinite number of the solutions like any problem with the use of approximate data. However, the set $Q_{\delta p}$ contains functions which can differ one from another on infinite value [9,16]. It is due to the reason that the operator A_p in the equation (5), as a rule, is completely continuous.

The formal definition of completely continuous operator can be found in any textbook on functional analysis [17]. In popular sense, these operators will transform "bad" functions to "good" ones. For example, the discontinuity functions can be transformed to continuous functions, frequently varying functions with the large amplitude to functions with small amplitude of variation, etc.

It follows that set $Q_{\delta,p}$ at any small value δ includes the functions with various norms and even with indefinitely various norm. Thus, the set $Q_{\delta p}$ includes the essentially different functions which are equivalent in sense of the solution of the equation (5). Therefore, the basic difficulty will be the selection of the concrete solution from infinite set of the various equivalent solutions. For this purpose it is necessary to involve some additional information. It is presented in [9,16].

For obtaining of the steady solutions of formulated above problems it is necessary to use the method of Tikhonov's regularization [9].

Let us consider the stabilizing functional $\Omega[z]$ which has been defined on set Z_1, where Z_1 is everywhere dense in Z [9]. Consider now the extreme problem:

$$\Omega[z_{\delta,p}] = \inf_{z \in Q_{\delta,p} \cap Z_1} \Omega[z],\ p \in D. \qquad (10)$$

It was shown that under certain conditions the solution of the extreme problem (10) exists, is unique and stable with respect to small change of initial data x_δ [9]. The function $z_{\delta,p}$ is named *the stable model of external load* after taking into account the only inaccuracy of experimental measurements. The solution of a problem (10) can be non-unique. For the purposes of mathematical simulation any such solution will be acceptable. Such model of external load can be used for mathematical simulation of initial system (2).

Still, there are no basis to believe, that the function $z_{\delta,p}$ will be close to real external load z_T. It is only good and steady model of external load [10,11,16].

However, such approximate solution can be interpreted in other way. The regularized solution can be treated as steadiest with respect to change of the factors which were not taken into account in mathematical model. These factors may include changes in structure of mathematical model of system, the influence, which were not taken into account, change of conditions of experiment etc. We can prove such interpretation of the solution.

At the synthesis of mathematical model of dynamic systems we will first of all take into account the factors which define a low-frequency part of change of state variables. First, it is due to the fact that this part of a spectrum is well observed during the experiment, as measuring devices do not deform it. Secondly, the high-frequency components of external loads as well as equivalent to them insignificant factors not taken into account quickly die away in process of distribution among inertial elements. Thus, factors of interactions, which were not taken into account and equivalent to them influences, change only high-frequency part of the approximate solutions. If the factors, which are not taken into account, change a low-frequency part of the solutions then it means that mathematical model of process is chosen incorrectly. In work [18] has been shown that the regularized solution $z_{\delta,p}$ represents result of high-frequency filtration of approximate solution. The greater degree of smoothing of the solution corresponds to greater error of initial data. Hence, the regularized solution $z_{\delta,p}$ can be interpreted as the function from set $Q_{\delta,p}$ which is the steadiest with respect to changes of factors, which are not taken into account. Such quality of the regularized solution is very important when it is used in mathematical modeling of real processes when the results of modeling are steady with respect to small changes factors, which are not taken into account and which are naturally present at any mathematical model of process.

For the numerical solution of an extreme problem (10) the discrepancy method was used [9]. The problem (10) was replaced by following extreme problem

$$\inf_{z\in Z_1} M^{\alpha}[A_p, B_p, z] =$$

$$= \inf_{z\in Z_1} \left\{ \left\| A_p z - B_p x_\delta \right\| + \alpha\Omega[z] \right\},\ p\in D, \qquad (11)$$

where parameter of regularization α was determined from a condition

$$\left\| A_p z_{\delta,p} - B_p x_\delta \right\| = b_0 \delta \text{, where } b_0 = \sup_{p\in D} \left\| B_p \right\|.$$

Previously has been proven, that the solution of problem (11) exists. In this case the analysis of a problem was reduced to the solution of the Euler's equation for functional (11) if the functional space U is Hilbelt space:

$$A_p^* A_p z_{\delta,p} + \alpha\Omega'[z_{\delta,p}] = A_p^* B_p x_\delta, \qquad (12)$$

where A_p^*, B_p^* - the associate operators to A_p; B_p; $\Omega'[z]$ - Frechet's derivative.

The approximate solution of the equation (12) on a uniform discrete grid was carried out by a numerical method.

The set of possible solution of equation (5) is necessary to extend to set $Q_{h_1,d_1,\delta}$ taking into account the inaccuracy of the operators A_p, B_p [9]:

$$Q_{h_1,d_1,\delta} = \{z : \left\| A_p z - B_p x_\delta \right\| \le$$

$$\le h_1 \left\| z \right\| + d_1 \left\| x_\delta \right\| + \delta b_0,\ b_0 = \sup_{p\in D} \left\| B_p \right\| \}$$

Any function from $Q_{h_1,d_1,\delta}$ causes the response of mathematical model coinciding with the response of investigated system with an error into which errors of experimental measurements and errors of a possible deviation of parameters of a vector $p \in D$ are included. A problem of a finding $z \in Q_{h_1,d_1,\delta}$ will be entitled by analogy to the previous one as a *problem of synthesis for a class of models* [5,10,16].

It should be noted that the set of the solutions of a problem of synthesis $Q_{h_1,d_1,\delta}$ at the fixed operators A_p from K_A and B_p from K_B in $Q_{h_1,d_1,\delta}$ contains elements with unlimited norm (incorrect problem) therefore the value $\delta b_0 + d_1 \left\| x_\delta \right\|_X + h_1 \left\| z \right\|_Z$ can be infinitely large. Formally speaking, such situation is unacceptable as it means that the error of mathematical modeling tends to infinity, if for the simulation of external load is used the arbitrary function from $Q_{h_1,d_1,\delta}$ as models of external load. Hence not all functions from $Q_{h_1,d_1,\delta}$ will serve as "good" models of external load.

The models of external load $z(t)$ in this case can be different. They will depend on final goals of mathematical modelling. For example, we can obtain model z_0 for simulation of given motion of system or process as solution of extreme problem [10,11,16]:

$$\Omega[z_0] = \inf_{z\in Q_{h_1,d_1,\delta}} \Omega[z]. \qquad (13)$$

The model of external load which is necessary for estimation from below of output of dynamic system (process) can be obtained as solution of the following extreme problem [10,11,16]:

$$\| A_b z_b \|_U = \inf_{z_{\delta,p}} \| A_p z_{\delta,p} \|_U , \qquad (14)$$

where $z_{\delta,p}$ is the solution of extreme problem (10).

Another model for estimation from above of output of dynamic system (process) can be obtained as solution the extreme problem [8,10]:

$$\| A_c z_c \|_U = \sup_{z_{\delta,p}} \| A_p z_{\delta,p} \|_U . \qquad (15)$$

As unitary model z_{op} we can call the solution of following extreme problem [8,10,16]:

$$\inf_{z_{\delta,a}} \sup_{A_p} \| A_p z_{\delta,a} - B_p x_\delta \|_U = ,$$
$$= \| A_{p_{op}} z_{op} - B_p x_\delta \|_U \qquad (16)$$

where $z_{\delta,a}$ is the solution of extreme problem (10) with $p = a,\ a \in D$.

To be more exact in approximate solution of extreme problems (13) - (16) the method of selection of special mathematical models was suggested in [6,18,20].

The comparative analysis of mathematical modeling with various known models of external loads and experimental data were presented in work [6,16]. The model of external load, z_{op}, better corresponds to new experimental observations.

In papers [8,10,16] another real calculations of models of external loads were performed which provide adequate results of mathematical simulation.

4 Conclusion

The problems of synthesis of adequate mathematical description of real dynamical system are considered in this paper. One of the possible solutions of above-mentioned problems is the choice of model of external loads adapted to dynamical system by identification method. The peculiarities of such approach were investigated. These problems are actually incorrect ones by their nature and that is way for their solution were used the regularization methods of Tikhonov. For the case when mathematical model are given approximately, different variants of choice of external loads models, which depend on final goals of mathematical modeling are considered. It can be as follows: modeling of given motion of system, different estimation of responses of dynamic system, modeling of best forecast of system motion, the most stable model with respect to small change of initial data, unitary model etc.

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