

# CONTROL OF DISCRETE EVENT SYSTEMS MODELLED BY PETRI NETS

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## Abstract

The control of a discrete event system implies the process of making the appropriate decisions to lead the DES to a desired behaviour that permit to achieve a set of objectives (usually related to financial profit, process time, utilization rate of the equipment, etc). Some of the decision problems that can be stated on industrial DES are associated to solution spaces whose size is determined by a combinatorial explosion. In these cases the computing time to solve the problem is a critical issue when a real application of the solution algorithm is searched. In this paper, it has been performed a systematic analysis on the characteristics of a DES where a decision problem can be stated, its translation into an undefined Petri net model and its representation by means of compound Petri nets and simple alternative Petri nets. As a result of this analysis, a promising research field is envisaged, which may permit to increase the efficiency of optimization algorithms developed so far, by means of the choice of the best representation of the Petri net model of the DES.

**Keywords:** Discrete event systems, undefined Petri nets, compound Petri nets, simple alternative Petri nets.

## Presenting Author's biography

Emilio Jiménez Macías studied Industrial Engineering (Computer Science, Electronics and Automation specialty) by the University of Zaragoza. After working several years in the industrial private sector (Researching and Developing Department head position), returned to the university, at the University of La Rioja, in 1997, where he works presently in the Electrical Engineering Department (Coordinator of the System Engineering and Automation Group). In 2001 he presented his PhD thesis about Industrial Automation. His research areas include factory automation, modeling and simulation, and industrial processes.



# 1 Introduction

## 1.1 Decision making in DES

The control of a technological system permits to restrict the behaviour of the system according to certain specifications. With the desired behaviour for the system some objectives are expected to be achieved; among them it can be considered maximal financial profit, minimal operation time, maximal quality in a manufactured product, etc. If there is a deviation in the planned behaviour for the system, a decision should be made to decide which action to take, among all the possible ones [1, 2, 3].

The choice of one action among all the possibilities may be an easy task, especially when the solution space has a reduced size [4]. Nevertheless, in some cases, the search for the appropriate decision to be made may require the development of a search process that might not be performed in a reasonable amount of time, according to the requirements of the system to be controlled. The development of techniques to find a good decision in a bounded amount of time is an important topic in the control of technological systems [5, 6, 7].

## 1.2 Undefined characteristics of a DES

On the other hand, in the field of industrial or logistic systems modelled as discrete event systems (DES), it is common that any system has some characteristics that should be defined prior to the specification in its operation or a change in it. Those undefined characteristics exist in every case in the design process of the system. They are related to what can be regarded as freedom degrees of a DES and are the necessary conditions that lead to a decision making process [8, 9].

It is the work of designers, engineers, managers, superintendents and supervisors to specify the best choice for those undefined characteristics in order to achieve the objectives of an organization. The solution to this problem is not always easy, and requires the consideration of multiple elements that can influence decisively the processes involved in the design and operation of a DES [10, 11]. In order to support the decision making in this area, a mathematical approach can provide with suggestions about the choices to make, taking into account the behaviour of the analysed system and using the calculation capabilities of computers. A formal statement of the decision problems must be posed in order to apply efficient methodologies to cope with this challenge in the chain supply and industrial fields.

## 1.3 Transformation of the problem of decision making

This paper will cope with the decision making in DES and the transformation of this process into a decision making based on a Petri net model of a DES. As a

result a definition of a new type of Petri net will be given: the compound Petri net, which is a very useful formalization in order to provide with efficient representation of a model of the original DES as a key element in simulation-based techniques to make decisions [12, 13].

## 2 Undefined discrete event system

An undefined DES can be described as a DES that contains some undefined characteristics that can be specified by means of the appropriate decisions [14, 15, 16]. As a consequence the result of solving a decision problem will lead to the application of this solution to the configuration of the undefined characteristics of the DES. If all the undefined characteristics of an undefined DES are specified, then the DES becomes a defined DES.

In industrial and logistic systems is a common objective for the decisions to obtain the best performance, which can be stated in different ways, involving the production rate, the time span, the WIP (work in process), the equipment utilisation, the benefit, etc.

Examples of undefined characteristics in an industrial facility can be the capacity of a storage buffer, the size of the pallets of raw materials, the assignment of tasks to the different machines in a certain interval of manufacturing time or the number of items a robot should pick up at a time. This definition also includes, as possible undefined characteristics, deeper uncertainties as the specific machinery that can be acquired and set up to perform a certain task in the operation of the facility, among all the alternatives that could be found in the market.

In the previous set of examples, two types of uncertainties have been mentioned: the ones related to the operation of the system, called operation characteristics, and the ones related to its design, named design characteristics.

An undefined DES requires the specification of a set of undefined characteristics in order to determine it in an unambiguous way and therefore to configure a defined DES with the optimal characteristics regarding the achievement of a certain objective. This fact makes the definition of a particular undefined DES a process of choosing one from a set of alternative DES that complies with the specifications of the problem. This statement will have an important consequence in the models that can be used as a representation of the original undefined DES. A feasible model for an undefined Petri net might be characterised by the inclusion, in a way or another, of an exclusiveness property that will reflect this nature of the alternative DES that can be chosen to specify the undefined DES. This property may lead, on one hand, to a broad set of different possible models for the undefined DES and, on the other hand, to the

particular nature of those models that make them differ from other models that have been researched so far.

## 2.1 The Petri nets as a modelling formalism

The logistic and manufacturing systems described as DES usually show a behaviour characterised by concurrence, parallelism, synchronisation and resource sharing. In order to apply an algorithmical methodology and to obtain a quantitative solution to the decision problem based on a DES it can be developed a model of the system able to describe, by means of an appropriate formalism, the constraints which configure the behaviour of the system. A Petri net (PN) is a paradigm that can cope with such a description in a double way: a graphical and intuitive one and an algebraic one that is suitable to perform a validation and a verification of the model and to be processed by automatic calculation in a computer [17, 18].

## 2.2 The modelling process of an undefined DES

The process to obtain a model of a DES based on the formalism of the PN has been broadly studied. There are two main approaches: bottom-up and top-down modelling. The undefined characteristics of a DES in a decision problem may have their quantitative representation in the Petri net model of the DES in what has been called undefined parameters. The Petri net model of a DES is a quantitative representation of the structure and behaviour of the original system. The parameters that configure the Petri net can be classified according to their function in the model. Any of those parameters can be undefined ones, giving place to an uncertainty that can be solved by means of a decision. Examples are the structural, marking, transition firing or interpretation parameters.

### Example 1

Let us consider an autonomous Petri net  $R = \langle Q, m_0 \rangle$ , where  $Q$  is defined by the incidence matrix:

$$W = \begin{pmatrix} \mathbf{t}_1 & \mathbf{t}_2 & \mathbf{t}_3 & \mathbf{t}_4 \\ -1 & \mathbf{w}_{12} & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & \mathbf{w}_{32} & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{matrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \\ \mathbf{p}_4 \end{matrix}$$

On the other hand, the initial marking  $m_0$  is defined as follows:

$$m_0^1 = (m_0^1 \ 0 \ 0 \ 0)$$

In this Petri net there are three undefined parameters. Two of them are structural ones, whereas the third one is an undefined marking parameter:

$$\alpha_1 = w_{12}, \alpha_2 = w_{32} \text{ and } \alpha_3 = m_0^1$$

They determine a set of undefined parameters of  $R$ , given by  $S_\alpha = \{ \alpha_1, \alpha_2, \alpha_3 \} = \{ w_{12}, w_{32}, m_0^1 \}$ .

As a general statement, the undefined parameters can be seen as quantitative representations of the undefined characteristics of the original system. The undefined parameters are integrated, with the rest of parameters of the PN, in the model of the system. The process of obtaining a set of undefined parameters in a Petri net model from a set of undefined characteristics of an original DES can be referred as translation. This process allows the implementation of numerical algorithms to calculate the most appropriate values for the undefined parameters as a solution of a certain decision problem. The result of the translation of an undefined Petri net into a Petri net model is an undefined Petri net.

The translation from the undefined characteristics of a DES into undefined parameters of a PN model of this DES is not restricted to find a set of undefined parameters of the Petri net that models one or several undefined characteristics. It is also necessary to convert the set of alternatives among which the undefined characteristic can be specified into a set of numbers that bound the values that the undefined parameters can take. It is also interesting to remember that it might not be enough to define an independent set of values as feasible solutions for every undefined parameter since the values of different parameters can be related defining a reduced set of valid combinations.

### Example 2

The set of feasible values for each one of the undefined parameters of the PN in the example 1 are:

$$S_{val\alpha 1} = \{ v_{(1,1)}, v_{(2,1)} \} = \{ 0, 1 \},$$

$$S_{val\alpha 2} = \{ v_{(1,2)}, v_{(2,2)} \} = \{ 0, 1 \} \text{ and}$$

$$S_{val\alpha 3} = \{ v_{(1,3)}, v_{(2,3)} \} = \{ 1, 2 \}$$

Eight different combinations of feasible values of the undefined parameters might be obtained:

(0,0,1); (0,0,2); (0,1,1); (0,1,2); (1,0,1); (1,0,2); (1,1,1) and (1,1,2)

Nevertheless, in this problem not all those combinations are allowed, but a subset of them:

$$v_1 = (0,1,1), v_2 = (1,0,2).$$

Hence, the set of valid combinations of feasible values for the undefined parameters of  $R$  is given by:

$$S_{val\alpha} = \{ v_1, v_2 \} = \{ (0,1,1), (1,0,2) \}$$

## 2.3 Undefined Petri nets

**Definition.** *Undefined Petri net*

An undefined Petri net is a triple  $R^U = \langle R, S_\alpha, S_{val\alpha} \rangle$ , where

$$R = \langle Q, m_0 \rangle \text{ is a marked Petri net,}$$

$S_\alpha = \{ \alpha_1, \dots, \alpha_n \} \neq \emptyset$  is the set of all undefined parameters of the Petri net  $R$ .

$S_{val\alpha}$  is the set of valid combinations of the feasible values for the undefined parameters in  $S_\alpha$ .  $\square$

**Remark 1:** The previous definition is applicable to an ordinary or generalized Petri net. For different abstraction levels or interpretations like priority Petri nets or synchronized/timed Petri nets it is necessary to add transition priorities, events, conditions, actions and/or delay times that correspond to each particular case. Any of those elements are represented by means of numerical parameters in the Petri net model, hence any of them can be an undefined parameter in a certain case.

**Remark 2:** If at least one undefined characteristic of the DES is modelled in the associated Petri net  $\Rightarrow$  the Petri net model of an undefined DES is an undefined Petri net.

This last *remark 2* refers to the fact that a discrete event system can be modelled by means of a Petri net according to different levels of detail. In some of these levels, the modelling process might not take into account the undefined characteristics of the original DES when they are not relevant enough for the purposes of the modelling process. In this case it a defined Petri net model can be obtained from an undefined discrete event system.

## 2.4 Solving algorithm

The important step of determining the set of possible values for the undefined parameters of a Petri net will contribute to define the size of the solution space of the decision problem because the valid combinations of the different values of the undefined parameters configure the feasible solutions of the problem as it will be stated later.

If a decision problem based on a DES is stated, and a Petri net model of the system is obtained, a numerical algorithm may be able to provide with appropriate values for all the undefined parameters. A further step in the solving process of the problem is to answer the question posed on the original decision problem, where a choice among different alternatives should be found to determine some of the undefined characteristics of the original discrete event system.

## 3 Compound Petri nets and simple alternative Petri nets

A compound Petri net is an immediate representation of an undefined Petri net, in the sense that it is a direct translation of the definition of undefined Petri net in formal language.

**Definition.** *Compound Petri net system*

A compound Petri net system, denoted  $R^c$ , can be defined as a seven-tuple  $\langle P, T, Pre, Post, m_0, S_\alpha, S_{val\alpha} \rangle$  where:

i)  $S_\alpha$  is the set of undefined parameters in  $R^c$  and verifies that  $\text{card}(S_\alpha) > 0$  (There exists at least one undefined parameter in  $R^c$ ).

ii)  $S_{val\alpha}$  is the set of feasible combinations of values for the undefined parameters of  $R^c$ .  $\square$

### Example 3

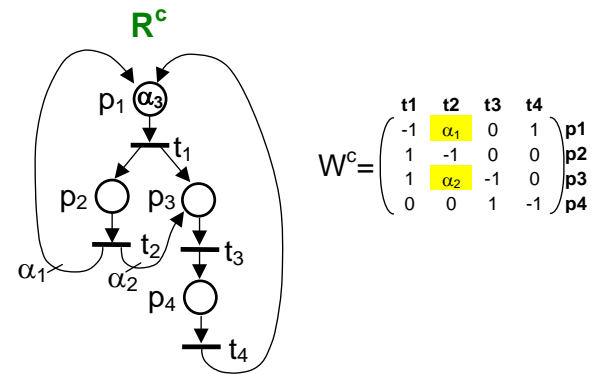


Fig. 1 Compound Petri nets.

A compound Petri net is one of the possible representations of an undefined Petri net, maybe the more direct and intuitive one. This representation leads to efficient implementation of optimization algorithms that can provide with the best solutions for a certain decision problem based on a DES.

Nevertheless, the compound Petri nets are not the only possible representation for an undefined Petri net. In fact, other representations can be found, which preserve the exclusiveness of all the alternatives that are feasible solutions for the undefined parameters of the associated undefined Petri net.

### Example 4

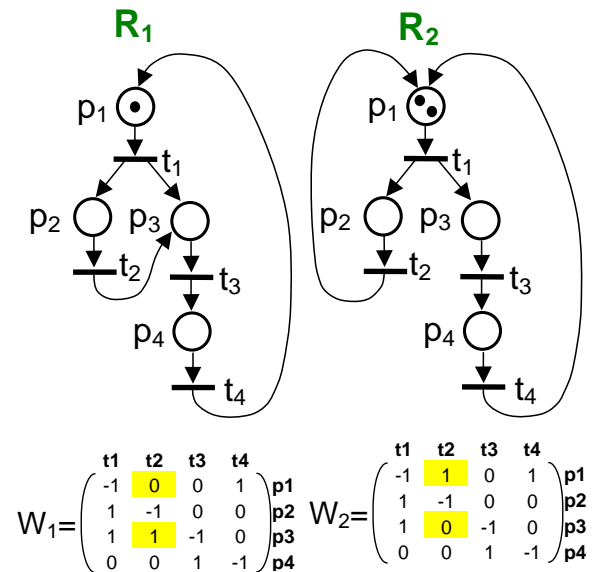


Fig. 2. Simple alternative Petri nets

There is one representation that appears commonly in the literature but it has not been an object of research by itself before this paper was developed. It is based in the concept of simple alternative Petri net. An intuitive description of this type of Petri nets consists of the specification of a different Petri net for every feasible combination of values for the undefined structural parameters of an associated undefined Petri net. It has to be considered that a structural parameter is that one which is an element of the incidence matrices of the PN.

## 4 Conclusions

In this paper it has been shown how it is possible to develop a systematic analysis of a DES that allows a decision making process to be developed on it, as well as its transformation into an associated undefined Petri net.

Another important issue on this paper is the possibility of obtaining different representations for the undefined Petri net. These different representations allow the development of diverse optimization algorithms whose efficiency can be directly related to the representation itself and the specific decision problem to be solved.

A compound Petri net and a set of simple alternative Petri nets are two of the possible representations of an undefined Petri net. An appropriate representation of an undefined Petri net may be more suitable for a certain type optimization problem than others. *In the paper the different representations of an undefined Petri net are analyzed accordingly to next Figure.* After an overview of the main possibilities that arise when an optimization problem is posed, a decision should be made on which one is the best representation to solve an optimization problem in an efficient way. This consideration is important since this kind of problems can be related to a combinatorial explosion that makes the exhaustive exploration of the state space unpractical. On the other hand, a compound Petri net seems to be a compact representation that allows to develop a solving algorithm that can, in a single stage, cope with complex systems with diverse uncertainties in it. Nevertheless, the need of an additional data structure to store the set of feasible combinations of values for the undefined parameters is a motivation for searching not only new representations for the undefined Petri nets but also the possibility of having more favourable representations by means of a compound Petri net. As a conclusion it is possible to state that the decision problem based on DES which are transformed into optimization problems based on undefined Petri nets allow the development of solving algorithms whose performance depend on the representation of the undefined Petri net considered for the statement of the problem.

## 4.1 Future research

The systematic study of the decision problems based on DES opens an interesting research field. In particular the analysis of the behaviour of different representations of a related undefined Petri net leads to the possibility of developing efficient optimization algorithms that may outperform the existing ones in the solution process of problems based on undefined Petri nets. This development might provide with a significant contribution to the decision making process in the industry with respect to a reduction in the computer requirements for developing optimization algorithms. In particular the computer time might be dramatically reduced in certain type of decision problems.

For this reason, the search for efficient representations of undefined Petri nets, as well as their performance in optimization algorithms regarding the characteristics of the considered decision problem can lead to a systematic methodology to choose an efficient algorithm to solve the problem, outperforming previous approaches found in the literature, where it has not been applied any systematic search for the best option.

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