

THE POSITION DETECTION SYSTEM OF THE AUTONOMOUS MODEL CAR GT-CAR

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Abstract

Gigatronik has developed GT-Car as a new experiment for laboratories in control theory, mechatronics, vehicle dynamics, or embedded software engineering at universities and technical colleges. GT-Car is an autonomous model car in the 1:10 scale driving on a conveyor belt with speeds up to $4.2\frac{\text{m}}{\text{s}}$. The control algorithms of GT-Car are developed in a model-based development process in MATLAB/Simulink. By one single mouse click, C-Code is generated from the Simulink model, compiled and flashed to the microcontroller of GT-Car, so that students can rapidly design and test control algorithms. Since GT-Car integrates the powerful microcontroller Infineon TriCore TC1796, computationally expensive advanced control algorithms can be applied in real-time. The focus of this paper is on the position detection algorithm of GT-Car that integrates infrared distance sensors and an inertial measurement unit. By processing the infrared distance signals as well as the acceleration and yaw rate signals of the inertial measurement unit, an onboard Kalman filter estimates the position of the car on the conveyor belt in real-time. In addition, the dynamical simulation model of GT-Car is presented that describes the vehicle dynamics in relative motion on a moving surface.

Keywords: Autonomous model car, Laboratory experiment, Position detection system, Kalman filter, Vehicle dynamics model, Model-based software development.

Presenting Author's Biography

Frank Tränkle received his Master of Science degree in Chemical Engineering at the University of Wisconsin in 1993 and his diploma in Technische Kybernetik at the Universität Stuttgart in 1994. In his doctoral thesis at the Universität Stuttgart he developed the modeling tool PROMOT for the object-oriented modeling and dynamical simulation of chemical processes. After his doctoral degree in 1999 he worked as a software engineer for hardware-in-the-loop test systems at ETAS GmbH in Stuttgart. He took over the lead of the LABCAR software development team at ETAS in 2001. Since 2006 Frank Tränkle is the director of the function development and simulation department at Gigatronik Stuttgart GmbH. His work focuses on model-based embedded software development, real-time vehicle simulation models, and hardware-in-the-loop testing.



1 Introduction

The model car GT-Car is a product of Gigatronik Stuttgart GmbH. GT-Car has been specifically developed for laboratories at universities and technical colleges. GT-Car is a mechatronic system that is built from standard RC-car components and state-of-the-art automotive electronic components (see Fig. 1).

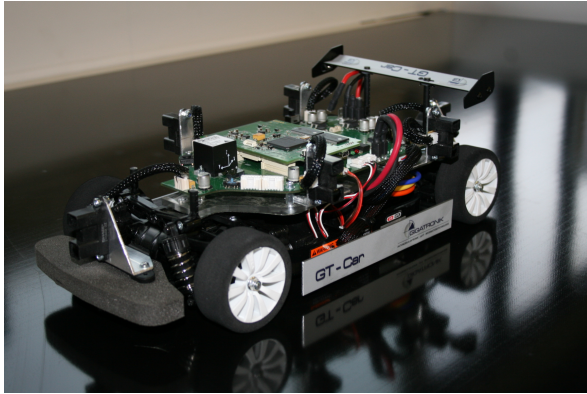


Fig. 1 GT-Car driving on the conveyor belt

The signal processing and control algorithms of GT-Car are developed in a model-based software development process that uses the established and widely spread tool chain MATLAB/Simulink with Real-Time-Workshop Embedded-Coder.

GT-Car is built around the electronic control unit GIGABOX gate XL [1] whose 32bit microcontroller is an Infineon TriCore TC1796 running at 150MHz. With the GIGABOX gate XL target, the Real-Time-Workshop Embedded-Coder generates C-code for this microcontroller. For the input of sensor signals and the output of actuator signals, the installation of the GIGABOX gate XL target includes a Simulink blockset. In laboratory experiments students learn to work with this tool chain which is also applied in the development of production code for automotive electronics.

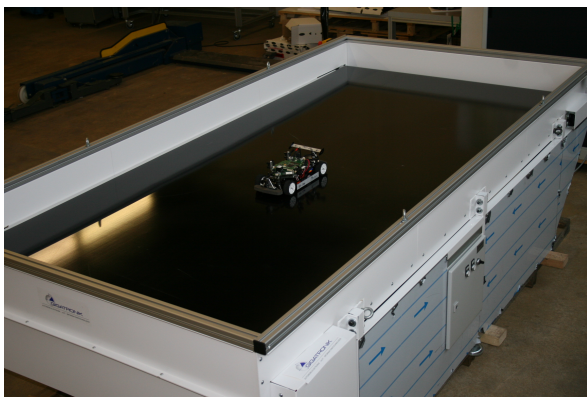


Fig. 2 The large-size conveyor belt

GT-Car drives on a conveyor belt that is available in different sizes. The largest configuration today has a

length of 300cm and a width of 150cm manufactured by Bertschinger GmbH&Co.KG in Aldingen, Germany (see Fig. 2). The belt speed can be set to any speed in the range of 0 to $4.2 \frac{m}{s}$. With its RS-232, RS-485 or CAN interfaces the belt can be remotely controlled from the user PC. Thus, velocity profiles in driving maneuvers can be played back in the laboratory.

Alternatively, GT-Car can drive in a fixed environment. For this, Gigatronik provides scenarios in the scale of 1:10 to realize and test parking assistance systems [2].

The on-board interfaces of GT-Car comprise CAN, USB, JTAG, and Bluetooth for the communication with the user PC or for inter-car communication. With the inter-car communication, platoon driving and passing maneuvers can be realized.

The focus of this paper is on the positioning system of GT-Car. The real-time computation of the car position on a conveyor belt of 300cm \times 150cm by on-board systems is a challenge. In order to achieve a resolution in the cm range, the position detection system incorporates a sensor cluster [3] of six infrared (IR) sensors Sharp GP2Y0A02YK and the inertial measurement unit (IMU) Analog Devices ADIS16362BMLZ.

The IR distance sensors measure the distance of the car to the edges of the belt. For this, the belt is surrounded by a rectangular frame with a height of 15cm. By processing the distance signals of the IR sensors as well as the acceleration signals and the yaw rate of the IMU, a Kalman filter [4, 3] estimates the position, the yaw angle and the velocity vector of the model car on the conveyor belt.

For the design of the position detection system, a single-track simulation model of the vehicle dynamics has been developed and applied. The major extension of this model compared to standard single-track models [5] is the relative movement of the car on a moving surface. The vehicle model can drive forward and backward relative to this moving surface.

The following Section 2 documents the governing equations of the vehicle dynamics model. Section 3 illustrates the position detection algorithm.

2 Vehicle Dynamics Model

For the design of the position detection system in virtual simulation, a single-track vehicle dynamics model is derived. The underlying assumptions of this model are:

- The car is one rigid body.
- The front and rear wheels are considered as one wheel each located on the center axis.
- The only degree of freedom of this rear center wheel is its rotation on the drive axle.
- The only two degrees of freedom of the front center wheel are its rotation and the steering.

- There is no vehicle pitch or roll.
- The center of mass is located on the center axis of the car.
- The vehicle has a rear-wheel drive and no front-wheel drive.

The objectives in designing the model are:

- The car can drive with arbitrary relative velocity vectors on the moving surface.
- The surface of the belt can drive with arbitrary speeds in the negative longitudinal direction.

2.1 Kinematics Model

The vehicle dynamics model consists of two parts: the kinematics model and the kinetics model. The kinematics model serves two purposes. Firstly, the kinematics model is applied as the system model of the Kalman filter of the position detection system in Section 3.3. Secondly, the kinematics model in combination with the kinetics model is applied for the design and test of the position detection system in virtual simulation. The kinematics and kinetics models are implemented as block diagrams in MATLAB/Simulink using continuous-time blocks and Embedded MATLAB. In the following the kinematics model is derived by applying the equations of relative motion.

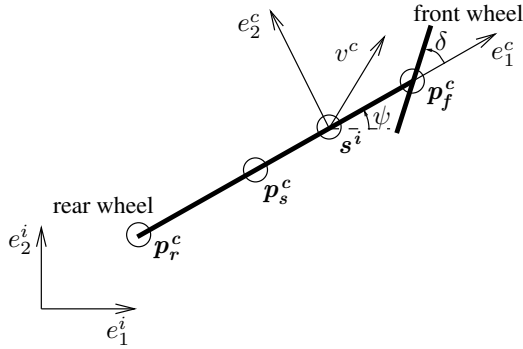


Fig. 3 The kinematics of the car. The IMU is located at the origin s^i of the fixed-body coordinate system. p_s^c is the center of mass.

As depicted in Fig. 3, the origin of the fixed-body coordinate system $\{e_1^c, e_2^c, e_3^c\}$ is at the position of the IMU. The origin of the inertial coordinate system $\{e_1^i, e_2^i, e_3^i\}$ is at the center of the belt.

The kinematics of the rigid body of the car in the fixed-body coordinate system is defined by the absolute velocity vector v^c and the angular velocity vector ω_{ci} .

- The absolute velocity vector v^c of the car at the position of the IMU in the fixed-body coordinate system is given by

$$v^c = \begin{pmatrix} v_1^c \\ v_2^c \\ 0 \end{pmatrix} \quad (1)$$

- The angular velocity of the car relative to the inertial system is defined as

$$\omega_{ci} = \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix} \quad (2)$$

where ψ is the yaw angle of the car.

- The position vector of the front wheel in the fixed-body coordinate system is

$$p_f^c = \begin{pmatrix} l_f \\ 0 \\ 0 \end{pmatrix} \quad (3)$$

- The position vector of the rear wheel in the fixed-body coordinate system is

$$p_r^c = \begin{pmatrix} -l_r \\ 0 \\ 0 \end{pmatrix} \quad (4)$$

- The position vector of the center of mass in the fixed-body coordinate system is

$$p_s^c = \begin{pmatrix} -l_s \\ 0 \\ 0 \end{pmatrix} \quad (5)$$

- The velocity vector of the IMU v^c is transformed to the velocity vector v_s^c of the center of mass by

$$v_s^c = v^c + \omega_{ci} \times p_s^c \quad (6)$$

For the computation of the wheel forces in the kinetics model the wheel speeds are needed.

- The wheel speeds at the surface contact points are

$$v_f^c = \begin{pmatrix} v_1^c - \omega_f r_w \cos \delta + v_b \cos \psi \\ v_2^c + l_f \dot{\psi} - \omega_f r_w \sin \delta - v_b \sin \psi \\ 0 \end{pmatrix} \quad (7)$$

$$v_r^c = \begin{pmatrix} v_1^c - \omega_r r_w + v_b \cos \psi \\ v_2^c - l_r \dot{\psi} - v_b \sin \psi \\ 0 \end{pmatrix} \quad (8)$$

where r_w is the radius of the wheels, v_b is the belt speed in the negative longitudinal direction, δ is the steering angle, and ω_f and ω_r are the angular velocities of the front and the rear wheels.

The velocity and position vectors of the IMU are defined by the following differential equations with corresponding initial conditions for the velocity and the position.

- The acceleration vector at the position of the IMU in fixed-body coordinates is given by

$$a^c = \frac{d v^c}{dt} + \omega_{ci} \times v^c \quad (9)$$

$$= \begin{pmatrix} \dot{v}_1^c - \dot{\psi} v_2^c \\ \dot{v}_2^c + \dot{\psi} v_1^c \\ 0 \end{pmatrix} \quad (10)$$

- The position of the IMU in the inertial coordinate system is given by

$$\frac{d\mathbf{s}^i}{dt} = \begin{pmatrix} v_1^c \cos \psi - v_2^c \sin \psi \\ v_1^c \sin \psi + v_2^c \cos \psi \\ 0 \end{pmatrix} \quad (11)$$

From Eq. (2), (10) and (11) the system model of the Kalman filter is derived in Section 3.3.

2.2 Kinetics Model

The kinetics model as part of the vehicle dynamics model consists of the conservative laws of motion and the phenomenological equations of the contact forces between the wheels and the belt surface.

- The momentum balance of the car in fixed-body coordinates is given by

$$m \frac{dv_{s1}^c}{dt} = F_{r1} + F_{f1} \cos \delta - F_{f2} \sin \delta + m \dot{\psi} v_{s2}^c \quad (12)$$

$$m \frac{dv_{s2}^c}{dt} = F_{r2} + F_{f1} \sin \delta + F_{f2} \cos \delta - m \dot{\psi} v_{s1}^c \quad (13)$$

The forces \mathbf{F}_r and \mathbf{F}_f are the contact forces of the rear and front wheels while m is the mass of the car and δ the steering angle. The velocity vector $\mathbf{v}_s^c = (v_{s1}^c, v_{s2}^c, 0)^T$ denotes the velocity of the center of mass and is defined in Eq. (6).

- The angular momentum balance is given by

$$\Theta \frac{d^2 \psi}{dt^2} = -F_{r2} \cdot (l_r - l_s) + (F_{f1} \sin \delta + F_{f2} \cos \delta) \cdot (l_f + l_s) \quad (14)$$

where Θ is the moment of inertia of the car.

- The contact forces are assumed to be proportional to the relative velocities of the wheel contact points which yields good numerical properties of the model for the numerical solution with variable-step solvers.

$$\mathbf{F}_f = -c_f \begin{pmatrix} 0 \\ v_f \\ 0 \end{pmatrix} \quad (15)$$

$$\mathbf{F}_r = -c_r \mathbf{v}_r^c \quad (16)$$

- The resulting side force of the front wheel is obtained by projecting the front wheel speed vector to the direction vector of the front axle

$$\begin{aligned} v_f &= \mathbf{v}_f^c \cdot \begin{pmatrix} -\sin \delta \\ \cos \delta \\ 0 \end{pmatrix} = \\ &= v_1^c \sin \delta + (v_2^c + l_f \dot{\psi}) \cos \delta \\ &= v_b \sin(\delta + \psi) \end{aligned} \quad (17)$$

- Finally, the rotation speed ω_r of the center rear wheel is defined by the following angular momentum balance

$$\Theta_r \frac{d\omega_r}{dt} = M - F_{r1} r_w \quad (18)$$

where Θ_r is the moment of inertia of the center rear wheel and M is the drivetrain torque on the rear axle.

In summary, the vehicle dynamics model consists of the system of ordinary differential equations (11), (12), (13), (14), (18) with corresponding initial conditions and the explicit algebraic equations (6), (7), (8), (15), (16), (17). The input signals of this equation system are the drivetrain torque M , the steering angle δ , and the belt speed v_b .

3 Position Detection System

The position detection algorithm is a software component of the driving algorithm of GT-Car that is developed in MATLAB/Simulink. This algorithm estimates

- the position $\hat{\mathbf{s}}^i = (\hat{s}_1^i, \hat{s}_2^i, 0)^T$,
- the yaw angle $\hat{\psi}$,
- and the velocity vector $\hat{\mathbf{v}}^i = (\hat{v}_1^i, \hat{v}_2^i, 0)^T$

of the car.

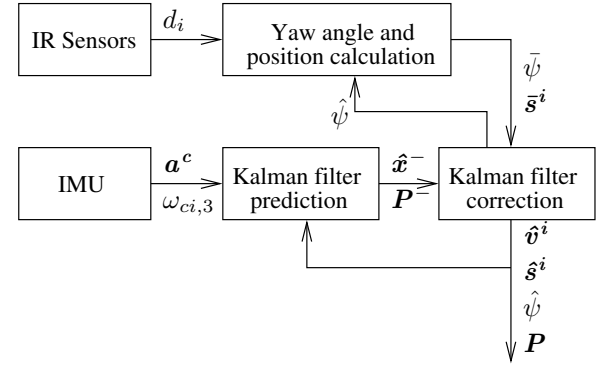


Fig. 4 The position detection algorithm

The challenge in the design of the position detection system is the low resolution of the IR distance signals. In the range of 100cm to 150cm the IR distance signals have a peak-to-peak noise amplitude of approximately 6cm. Further, the IR sensors sample the distance signals in a sampling rate of approximately 40ms. The requirement, however, for the position detection system is a resolution in the cm range at a sampling rate below 5ms.

For these reasons, a sensor cluster of six IR distance sensors and one inertial measurement unit (IMU) is applied. The position detection algorithm reads in the distance signals d_i of the six IR distance sensors $i =$

1, 2, ..., 6 on the one side. On the other side, the IMU signals yaw rate $\omega_{ci,3}$ and the longitudinal and lateral acceleration $\mathbf{a}^c = (a_1^c, a_2^c, 0)^T$ are processed.

As illustrated in Fig. 4, the position detection algorithm is subdivided into the following processing steps:

1. The yaw angle is computed from the IR distance signals.
2. The car position is computed from the yaw angle and the IR distance signals.
3. The Kalman filter predicts the velocity, the position, and the yaw angle by processing the IMU signals.
4. The Kalman filter corrects the predicted velocity, position and yaw angle signals by comparison to the position and yaw angle computed from the IR distance signals.

The output signals of the Kalman filter are the velocity, the position, and the yaw angle of the car.

3.1 Computation of the Yaw Angle from the IR Distance Signals

The yaw angle is defined by geometrical relations of pairs of IR distance signals. At first the yaw angle is individually computed for four pairs. Then the average value of these individual values is computed to reduce the signal noise.

- Right-side distance signals $i \in \{1, 2\}$

$$\tan \psi_1 = \frac{d_1 - d_2}{p_{1,1} - p_{2,1}} \quad (19)$$

- Left-side distance signals $i \in \{3, 4\}$

$$\tan \psi_2 = \frac{d_3 - d_4}{p_{4,1} - p_{3,1}} \quad (20)$$

- Front-right and front-left distance signals $i \in \{1, 3\}$

$$\cos \psi_3 = \frac{l_2}{d_1 - p_{1,2} + d_3 + p_{3,2}} \quad (21)$$

- Rear-right and rear-left distance signals $i \in \{2, 4\}$

$$\cos \psi_4 = \frac{l_2}{d_2 - p_{2,2} + d_4 + p_{4,2}} \quad (22)$$

- Average value of yaw angle

$$\bar{\psi} = \frac{1}{4} \sum_{i=1}^4 \psi_i \quad (23)$$

The rectangular dimensions of the belt are given by its length l_1 and its width l_2 . The vectors $\mathbf{p}_i^c = (p_{i,1}, p_{i,2}, 0)^T$ denote the position coordinates of the six IR distance sensors in the fixed-body coordinate system on the car as depicted in Fig. 5. The coordinate origin is the position of the IMU on the car.

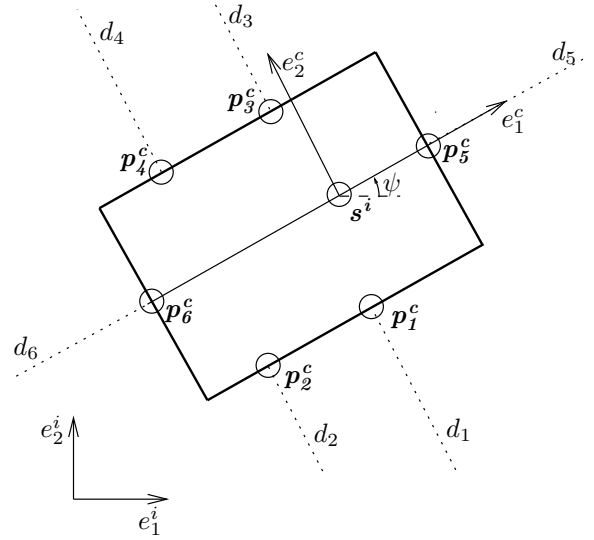


Fig. 5 The locations of the IR distance sensors. Again the origin of the fixed-body coordinate system \mathbf{s}^i is the position of the IMU.

3.2 Computation of the Car Position from the IR Distance Signals

The longitudinal and lateral position of the car in inertial coordinates $\bar{\mathbf{s}}^i = (\bar{s}_1^i, \bar{s}_2^i, 0)^T$ is computed from the yaw angle and all six IR distance signals. The yaw angle $\hat{\psi}$ estimated by the Kalman filter rather than $\bar{\psi}$ is taken as an input signal to this computation because $\hat{\psi}$ has a higher accuracy than $\bar{\psi}$ in Eq. (23). The lateral position is computed from the right-side and left-side distance signals, whereas the longitudinal position is computed from the front and rear distance signals.

- Right-side distance signals $i \in \{1, 2\}$

$$\begin{aligned} s_{i,2}^i &= d_i \cos \hat{\psi} \\ &- \left[\frac{l_2}{2} + (p_{i,1} \sin \hat{\psi} + p_{i,2} \cos \hat{\psi}) \right] \end{aligned} \quad (24)$$

- Left-side distance signals $i \in \{3, 4\}$

$$\begin{aligned} s_{i,2}^i &= -d_i \cos \hat{\psi} \\ &+ \left[\frac{l_2}{2} - (p_{i,1} \sin \hat{\psi} + p_{i,2} \cos \hat{\psi}) \right] \end{aligned} \quad (25)$$

- Front distance signal $i = 5$

$$s_{5,1}^i = -(d_5 + p_{5,1}) \cos \hat{\psi} + \frac{l_1}{2} \quad (26)$$

- Rear distance signal $i = 6$

$$s_{6,1}^i = (d_6 - p_{6,1}) \cos \hat{\psi} - \frac{l_1}{2} \quad (27)$$

- Average values of longitudinal position \bar{s}_1^i and lateral position \bar{s}_2^i

$$\bar{s}_1^i = \frac{1}{2} \sum_{i=5}^6 s_{i,1}^i \quad (28)$$

$$\bar{s}_2^i = \frac{1}{4} \sum_{i=1}^4 s_{i,2}^i \quad (29)$$

3.3 Kalman Filter

As the system model for the Kalman filter the kinematics model is applied, which has been derived in Section 2.1. The Kalman filter estimates the velocity and position vectors in inertial coordinates, whereas the IMU measures the acceleration in fixed-body coordinates. By considering Eq. (2) and applying the coordinate transformation on Eq. (10) and (11), the system model is obtained:

$$\underbrace{\frac{d}{dt} \begin{pmatrix} v_1^i \\ v_2^i \\ s_1^i \\ s_2^i \\ \psi \end{pmatrix}}_{\dot{x}} = \underbrace{\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}}_{A^*} \cdot \underbrace{\begin{pmatrix} v_1^i \\ v_2^i \\ s_1^i \\ s_2^i \\ \psi \end{pmatrix}}_x + \underbrace{\begin{pmatrix} a_1^c \cos \psi - a_2^c \sin \psi \\ a_1^c \sin \psi + a_2^c \cos \psi \\ 0 \\ 0 \\ \omega_{ci,3} \end{pmatrix}}_{g^*(x,u)} ; t > 0 \quad (30)$$

$$\underbrace{\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}}_y = \underbrace{\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}}_H \cdot \underbrace{\begin{pmatrix} v_1^i \\ v_2^i \\ s_1^i \\ s_2^i \\ \psi \end{pmatrix}}_x ; t \geq 0 \quad (31)$$

$$x(0) = (0 \quad 0 \quad s_{0,1}^i \quad s_{0,2}^i \quad \psi_0)^T \quad (32)$$

The IMU signals a_1^c , a_2^c and $\omega_{ci,3}$ are the input signals of this system model. The acceleration signals a_1^c and a_2^c are measured in the fixed-body coordinate system of the car. The IMU signal $\omega_{ci,3}$ is the yaw rate of the car. Eq. (30), (31) and (32) define the velocity vector v^i , the position s^i , and the yaw angle ψ in the inertial coordinate system.

Time discretization of the signals and differential equations by the Euler Forward method at time points t_k

$$x_k = x(t_k) \quad (33)$$

$$u_k = u(t_k) \quad (34)$$

$$A = A^* \Delta t \quad (35)$$

$$g(x_k, u_k) = g^*(x_k, u_k) \Delta t \quad (36)$$

yields the prediction step of the Kalman filter

$$\hat{x}_k^- = A \cdot x_{k-1} + g(x_{k-1}, u_{k-1}) \quad (37)$$

$$\hat{y}_k^- = H \cdot \hat{x}_k^- \quad (38)$$

$$P_k^- = A \cdot P_{k-1} \cdot A^T + Q \quad (39)$$

for time steps $k > 0$. As inertial conditions of the Kalman filter a non-moving car in the belt center is assumed:

$$\hat{x}_0 = (0 \quad 0 \quad 0 \quad 0 \quad 0)^T \quad (40)$$

$$P_0^- = 0 \quad (41)$$

In the correction step the predicted position and yaw angle are compared to the position \bar{s}^i and the yaw angle ψ computed from the IR distance signals in Sections 3.1 and 3.2. The correction step is given by

$$K_k = P_k^- \cdot H^T \cdot (H \cdot P_k^- \cdot H^T + R)^{-1} \quad (42)$$

$$\hat{x}_k = \hat{x}_k^- + K_k \cdot (y_k - \hat{y}_k^-) \quad (43)$$

$$P_k = (I - K_k \cdot H) \cdot P_k^- \quad (44)$$

for $k \geq 0$.

Hence, the estimated position \hat{s}_k^i and yaw angle $\hat{\psi}_k$ are available as control variables for the longitudinal and lateral vehicle dynamics controllers. On the given real-time hardware GIGABOX gate XL, the total software including the position detection algorithm and the vehicle dynamics controllers runs with sampling rates down to 2.5ms. An advantage of this algorithm is that no numerical differentiation of the position signals has to be applied in the D parts of the PID vehicle dynamics controllers, since the time-derived position signals \hat{v}_k^i are directly given as system state variables of the Kalman filter.

4 Conclusion

The autonomous model car GT-Car has been developed for the design and implementation of control algorithms in the areas of vehicle dynamics and driver assistance systems. The tool chain MATLAB/Simulink with the Real-Time-Workshop Embedded-Coder allows the rapid development of control algorithms in short development cycles suitable for laboratory experiments. With the Simulink blockset for the electronic control unit GIGABOX gate XL all relevant I/O signals of GT-Car can be accessed in Simulink.

The on-board sensor cluster of infrared distance sensors and the inertial measurement unit together with the position detection algorithm achieves the required resolution of 1cm at a sampling rate of 2.5ms. The single-track vehicle simulation model proves to be a suitable platform for the design and test of the position detection and control algorithms of GT-Car.

With little effort, the algorithms, which have been developed in virtual simulation in MATLAB/Simulink, have been readily implemented and applied on the real

GT-Car. For this, the Real-Time-Workshop Embedded-Coder automatically generates the C code and execution binary file for the GIGABOX gate XL from the Simulink model.

Future work focuses on the development of additional on-board camera-based sensor systems for the position detection of other vehicles and obstacles on the belt. Further emphasis is on the realization of platoon driving and automated passing maneuvers.

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