

# A FUZZY PETRI NET MODEL FOR ESTIMATION OF TRAIN DELAYS

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## Abstract

Even with the best timetable, trains often operate with delays. Planned duration of running and dwelling can be exceeded and that creates primary delay. Primary delay of a train can cause delays of other trains, knock-on delays. Estimation of train delays is important for timetable creation, trains dispatching, infrastructure planning etc. Many factors influence and cause trains delays and it is very difficult to estimate and describe their relations. This paper presents simulation model for train delays estimation based on Fuzzy Petri Nets (FPN). Fuzzy logic system incorporated in FPN uses experts (train dispatchers, operators etc.) knowledge for defining fuzzy sets and fuzzy rules and thus transforming their expertise into a model for train delay calculation. Petri Nets simulation model describes traffic processes in a railway system. Trains are *tokens*, track sections are *places* and *transitions* are discrete events of train moving and interlocking principles. High Level Petri Nets (HLPN) model has properties of hierarchy, color and time. Train delays are calculated in a simulation model by a Fuzzy Petri Net subsystem. Simulation model can be verified and validated by animation of train movement and graphically by train's time-distance graph. Results of simulation are exported to database for additional data mining and comparative analysis. Model is tested on a part of Belgrade Railway Node.

**Keywords:** Fuzzy Petri Nets, Train delays, Railway simulation.

## Presenting Author's biography

Sanjin Milinković received his Dipl. Ing. degree in traffic and transportation engineering in 2001. and his MSc degree in 2007., from University of Belgrade, Faculty of Transport and Traffic Engineering. He is currently working on a PhD thesis and his research interests include analysis of railway systems by modeling, simulation and operations research methods. Hi is currently working as a teaching associate with University of Belgrade, Department for railway exploitation.



## 1 Introduction

Train delays are one of the most used parameters for decision making on timetables and infrastructure problems. Also, they have very important place in train dispatching and railway traffic operations. Variations in train delays are from day to day, or even in same day, from hour to hour, and unpredictability of delays makes efficient planning of railway operations very difficult even for short period of time.

Primary disturbances are delays of trains caused by external stochastic disturbances. When primary delays develop inside the observed network they are called original delays. If buffer times between trains are less than primary disturbance then this is propagated to other trains. Primary delay of one train can cause delays on other trains and create knock-on or secondary delays. It is very difficult to calculate and predict secondary delay because they depend on length of primary delays, trains timetable, infrastructure (single or double track, station layouts, interlocking etc.) etc. Cause of primary delays are often technical failures, lower than scheduled running speed, prolonged alighting and boarding times of passengers, and bad weather conditions. Distribution of the primary delays can be obtained by a statistical analysis of existing empirical data. The knock-on delays of trains often occur during their approach or departure at stations, since the crossing or merging of lines and platform tracks are in most cases the bottlenecks in highly used railway networks. Yuan and Hansen [1], proposed an analytical stochastic model of propagation delays in the stations for calculation of secondary delays. Landex, Kaas, and Hansen, [2] defined unexpected waiting time, primary delay and the secondary delay and present the analytical method for their calculation.

In literature three different approaches can be found. Problem can be solved by (Mattsson, [3]): analytic methods, microsimulation methods and statistical analyses based on empirical data.

Analytical model is based on using the Queuing theory and does not require a lot of input data and usually apply some form of simplifying assumptions of system. Queuing models estimate the total (average) waiting time of trains at platform tracks or junctions and are applied in the course of strategic planning to evaluate the impact of increasing train frequencies and modifying infrastructure and train characteristics on the waiting time (Schwanhauber [4]; Huisman et al., [5]). Simulation models are detailed representation of a railway system, where different trains interact with each other and with the infrastructure. They require data about the infrastructure, the performance of the trains and about the timetable [6]. If one of these data is unknown it is necessary to make assumptions and then results depend on the quality of input data. Microscopic simulation tools can be used to model the propagation

of train delays in large railway networks, but require extensive work to model the infrastructure topology, signaling and timetables (simulation software RailSys [7] and OpenTrack [8]).

Statistical analysis is mostly used for modeling the occurrence of primary delays. Observed data on the delays could be used to establish empirical relationships between capacity utilization and secondary delays, given the prevailing level of primary delays. This can be applied in railway systems that are well regulated and operate in stable conditions. In systems where there are many possible sources for disruptions and a relatively high probability that external influences induce primary delays it is difficult to find a relationship that would calculate train delays.

Fuzzy logic is proved to be a good mathematical tool for modeling traffic processes that are distinguished by subjectivity, uncertainty, ambiguity and imprecision (Teodorovic, [9]). Many authors use advantage of predictive modeling systems with fuzzy logic. Fay [10] used fuzzy system as a dispatching support system for use in railway operation control systems. The model is defined as a fuzzy Petri net model that combines expert knowledge of fuzzy systems and graphical power of Petri Nets, making the model easy to design, test, improve and maintenance. Cheng and Yang [11] proposed fuzzy Petri Net model that will use professional knowledge of a dispatchers to create database rules to be applied for testing the system in case of disorder.

For most technical systems, a problem solution exists as the experience of experts who have dealt with these problems for a long time. Proposed fuzzy logic model for calculation of train's delays use the expertise, experience and knowledge of railway personnel who directly participate in regulating the traffic in system. Data from personnel interviews, timetable information is used to define parameters of a fuzzy system in a Fuzzy Petri Net for forecasting trains delays. This enables that parameters of fuzzy logic system can be different for any specific case. Knock-on delays cannot be directly calculated and they are calculated from a simulation results.

Simulation tool must be able to make a model that will incorporate all interlocking and operating rules and data. Petri Nets are tool for graphical and mathematical modeling of various systems. High Level Petri Nets - HLPN (timed, colored, stochastic and hierarchical) are tool that can model complex system and have a good graphical presentation of model. Simulation models for analyzing railway systems can be found in literature in past 20 years. Authors have presented models for analyzing various railway systems with focus on train delays.

Basten, Roland and Voorhoeve, [12] created a simulation model for the analysis of interlocking specification using colored Petri Nets in software

Expect. Van der Aalst and Odijk [13] proposed the interval timed colored Petri Nets (ITCPN) for modeling and analysis of railway stations, where train delay is specified by an upper and lower bound, i.e. an interval. Daamen, Goverde, and Hansen [14] developed a colored Petri Nets (CPN) tool for route conflict identification and estimation of knock-on delay.

## 2 Petri Nets

Petri nets are a mathematical tool for modeling used for analysis and simulation of concurrent systems [15]. The theory of Petri nets is based on a mathematical theory of bipartite graphs. A bipartite graph (or bigraph) is a graph which nodes can be divided into two disjoint sets  $V_1$  and  $V_2$  such that every edge connects a node in  $V_1$  to one in  $V_2$ ; that is, there are no two identical nodes in the same set. Petri net is one of several mathematical descriptions of discrete distributed systems. The system is modeled as a bipartite directed graph with two sets of nodes: the set of places which represent state or system objects and the set of events or transitions that determine the dynamics of the system. High Level Petri Nets (HLPN) is defined as follows.

A HLP-net is a structure  $HLPN = (S; T; C; C; Pre; Post; M_0)$  where

- $S$  is a finite set of elements called *Places*
- $T$  is a finite set of elements called *Transitions* disjoint from  $S$  ( $S \cap T = \emptyset$ )
- $C$  is a non-empty finite set of *types*
- $C : S \cup T \rightarrow C$  is a function used to type places and determine transition modes
- $Pre; Post : TRANS \rightarrow \mu PLACE$  are the pre and post mappings with

$$TRANS = \{(t, m) \mid t \in T, m \in C(t)\}$$

$$PLACE = \{(s, g) \mid s \in S; g \in C(s)\}$$

- $M_0 \in \mu PLACE$  is a multiset known as the initial marking of the net.

Places are represented with circles, and transitions are represented with rectangles. Transition of a system from one state to another occurs when event happens, where an event can also be the moment when time period in some state expires.

Number of tokens in place  $p_i$  is equal to the value of marking  $\mu_i$ . Input and output functions are represented with directed lines – arrows.

## 3 Petri Net model

Modeling the propagation of train delays always focuses on a specific track layout, signaling and train protection system and timetable design. Based on primary delays at the system boundaries the knock-on delays can be estimated by a Petri Net simulation model. Model of a part of railway network can show effects of primary disturbances on train movements. In ideal situation train operates without any delays. Propagation of primary delays in model will cause route conflicts and waiting for connections. Experimenting with model can help in analyzing secondary delays and overall performance of system. In Petri net model *places* represent track sections, *transitions* represent conditions for train movement, and *tokens* represent trains. Hierarchy of the model enables defining insulated track section as a subsystem or module. Insulated section can be block section, switch section and station track section, but more detailed description is needed (regarding position of a section relative to signals, stations and junction). A module is defined for each distinctive section. Model is created by positioning and connecting modules according to the railway line section plan. Although this approach requires more time for initial programming, it allows using predefined modules for modeling systems with similar processes (modeling of traffic processes in the station, on an open railway line etc.). The junction system model is created in software ExSpect v6.41 [16], where High Level Petri nets are with following dialects: hierarchical, timed, stochastic, colored [17].

Basic module for creation of model is module of track section. Other modules are similar and use the same principles of modeling interlocking principles, safety and signaling systems. Difference is in number of connecting input and output pins (for modules of switch) and in some additional rules for train movement within a station area. Module for generating trains uses timetable data imported from an external database for generating tokens (trains). Each token is loaded with an information about train it represent (train number, category, time of entering into the system, train route etc.). Tokens leave the module when the simulation clock and time of train departure match. Module of block section (Fig. 1) represents a block section on an open track. The module contains *places*, *transitions*, storages for storing parameters and objects for connecting with other modules. Transitions in module is enabled or not for entering and leaving the section based on the storage data. Storages contain information about state of the connected adjacent sections and signals and simulation clock. When the transition *trainin* enables firing, token are placed in *sectionbusy*. Instantaneously, information about the occupancy of the section is sent to the previous two sections and to the signal. Token remains in place until given conditions defined in the transition *trainout* are met:

simulation clock advances for amount of time needed for train to cross section - section occupancy time (train traveling time on section), next section is not occupied, and signal does allow further movement.

When the conditions are met, transition is allowed; the signal is set to allow train movement to next section; token leaves the section module; additional token is firing to the place *sectionfree*; and information about leaving the section are sent to connected modules. The purpose of storages in module is keeping data about section state. Data is used in transition processor for imposing logical conditions and for calculation of train journey time on section. The other type of storages (*sectionstate*) serves for gathering data generated when transition fires. These storages have information about the state of signals and sections occupancy.

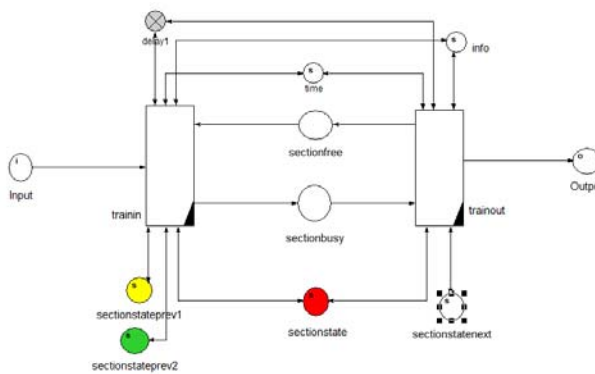


Fig. 1 Module of block section

External databases stores input and output data. The simulation program sends data about movement of each train through the model, as well as data about section state (occupancy of each section). The database is customized for creating quick reports based on queries and for filtering data by train, section, signals or train delay time in the model. With programmed macros, data from the database is used for creating train diagram (time – distance diagram) with vertical axis representing section lengths and horizontal axis representing time.

Animation of the simulation run is done in the simulation program itself, animating section states in the model. During the simulation, parameters (from storages) of each section change when a train enters the section. These numerical data are used for animation of sections.

#### 4 Fuzzy Petri Nets

There are different constructions of Fuzzy Petri Nets (FPN). As mentioned in Chapter 2, Petri Nets (PN) are constructed by using 4 types of objects, transitions, places, tokens and arcs. All of these objects may be fuzzified (fuzzy token, fuzzy place, fuzzy transition and fuzzy arc) [18]. Fuzzy token is a generalization of the token by giving it a truth value for belonging to a place. Token has a linguistic value, defined as a

membership function for a linguistic variable. This function also determines the degree of membership in a particular place, or the truth value of that proposition.

Modeling of fuzzy systems with classical Petri Nets or HLPN presumes that elements are redefined so it could presents fuzzy information and that structural and functional elements are defined for specific features of fuzzy system [19,20].

The set of IF-THEN rules, which forms linguistic description in fuzzy control system can is:

Rule 1:

IF  $X_1$  is  $A_{11}$  AND ... AND  $X_n$  is  $A_{1n}$  THEN  $Y$  is  $B_1$   
 .....

Rule m:

IF  $X_1$  is  $A_{m1}$  AND ... AND  $X_n$  is  $A_{mn}$  THEN  $Y$  is  $B_m$

These rules can be graphically presented by FPN with places, transitions and arcs (Fig. 2).

Fuzzy inference process comprises of five parts: fuzzification of the input variables, application of the fuzzy operator (AND or OR) in the antecedent, implication from the antecedent to the consequent, aggregation of the consequents across the rules, and defuzzification [21].

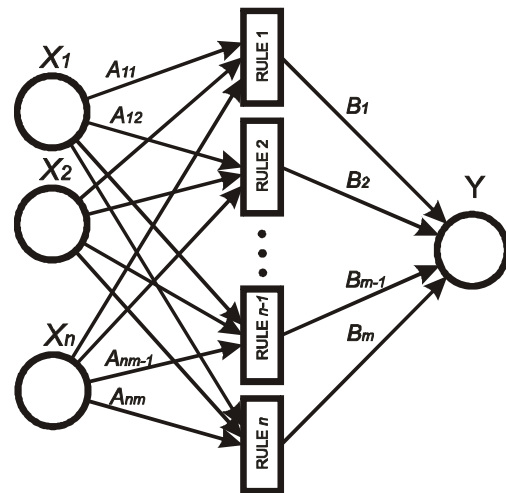


Fig. 2 Graphical presentation of rule base in Fuzzy Petri Nets

The fuzzification module enables transition of each input variable and maps the crisp input signal to a corresponding fuzzy value. Fuzzification of the input is by a function evaluation. Input value  $x_0$  is evaluated by input variables defined with fuzzy set  $A$ , and  $\mu_A(x)$  is calculated. Each input is fuzzified over all the qualifying membership functions required by the rules. Next, after the inputs are fuzzified, if the antecedent of a given rule has more than one part, the fuzzy operator is applied to obtain one number that

represents the result of the antecedent for that rule. This number is then applied to the output function. The fuzzy AND operator selects the minimum of the input values. Every rule has a weight, which is applied to the number given by the antecedent. In model weight of every rule is 1. Then, implication method is implemented producing a fuzzy set represented by a membership function. The consequent is reshaped using a function associated with the antecedent (a single number). Implication is implemented for each rule. Method for implementation is MIN (minimum). Aggregation function then combines results from all rules into a single fuzzy set. Aggregation method in FPN is MAX (maximum). Finally, last step, defuzzification converts aggregated fuzzy output set into a crisp number (single number). Authors have tested various defuzzification methods (centroid or center of gravity, middle of maximum and bisector) in previous research. Tests have shown that these methods of defuzzification do not give results that are expected and that they deviate from real system observed data. Center of Gravity (COG) method gives crisp numbers that are precise for relatively large values of train delay, but deviation appears for smaller delays. COG produce values that cannot be used for lowest output value (very small delay). Since method produce center of gravity for aggregated are it is not possible to produce zero value for train delay.

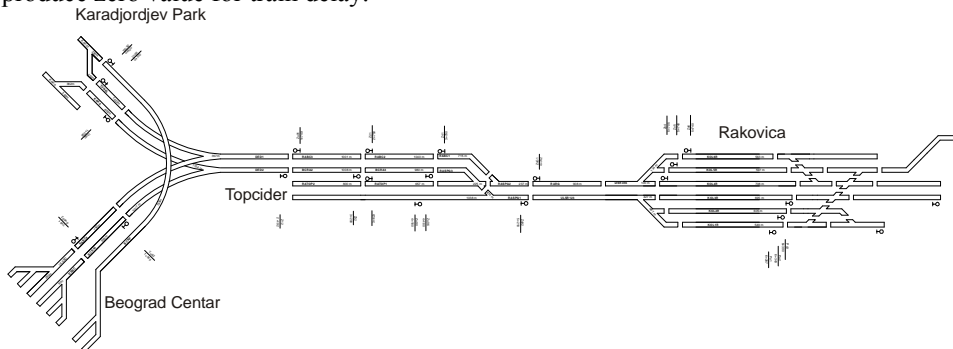


Fig. 3 Plan of the block section for a PN model

## 5 Application of a model

Model is tested on a part of Belgrade railway node. The boundaries of the model are stations Belgrade Center, Topcider, Rakovica and Karadjordjevi Park (Fig. 3.). Model is defined by infrastructure data of sections and railway signals layout plan, and with timetable data from year 2008. There are three train categories in model: freight, regional (regional and suburban) and passenger (long distance passenger trains). Train movement is modeled by section occupancy time which depends on train length, train acceleration and deceleration, as well as its maximum speed on the section. Spacing of train is by Automatic Block System [22]. The station sections have additional dwell time in stations. The physical occupancy time of the section is the time from

Therefore, authors suggest modified COG method where marginal values for small delays are calculated by linear membership function of the leftmost fuzzy set. For example, this enables that when output fuzzy set  $B_1$  has probability of  $\mu_B(x)=1$ , crisp value is 0, and for  $\mu_B(x)=0$ , crisp value is value of right limit of membership function.

FPN presented in this paper is developed for calculation of train delays on a section of double-track network between three stations. Depending on a type of system that is modeled, different combinations of input parameters can be applied. This FPN model has three input parameters: train category, timetable influence and train traveled distance. Influence of infrastructure parameters is not considered in this model since these parameters have same effect on all trains. The train category and probability of train delay are highly dependent. Input parameter called timetable influence on train delays comprises of various effects: time of operating, types of locomotives, local conditions, technological solutions, principles for safety and signaling, weather conditions etc. Long distance traveled increase the probability of train delay.

moment when first shaft enters the section, to the moment when the last shaft leaves the same section. Additional delay of trains in sections can occur when the next block section is occupied by the previous train (knock on delay). The total occupancy time is the period in which the section is occupied with trains in movement. Beside physical occupancy, it accounts for the time in which the section is reserved for train route. Parameters in the model are defined for the section and for token/train. Train parameters are defined in "colored" tokens that carry information on time of departure, train category, occupation time from last section, and time of enter and depart from last section. Time data stored in token is dynamically changing as train move from section to section.

Fuzzy Petri Net model for delay calculation is first subsystem that token enters in a model. After the

delay is calculated train enters to first section (section of a station track) in a model when simulation clock time equals time of train departure plus delay time. Parameters of a FPN are defined in collaboration with traffic dispatchers, operators and experts familiar with functioning of the system. Their knowledge and experience, as well as train delay statistics, are used for defining input variables, rules base and output variables. FPN is defined with three input variables: train category (Fig. 4), timetable influence on a train delay (Fig. 5) and train traveled distance (Fig. 6).

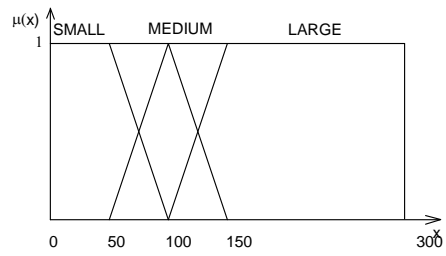


Fig. 6 Membership functions of input parameter - distance

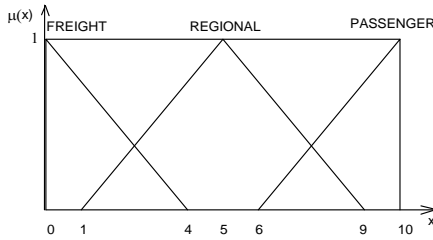


Fig. 4 Membership functions of input parameter – train category

The membership function for output variable train delay is defined with 5 fuzzy sets: very small ( $\mu_{VS}$ ), small ( $\mu_S$ ), medium ( $\mu_M$ ), high ( $\mu_H$ ) and very high delay ( $\mu_{VH}$ ) (Fig. 7).

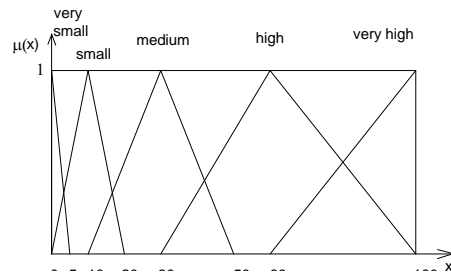


Fig. 7 The membership function of fuzzy set of output variable ( train delay )

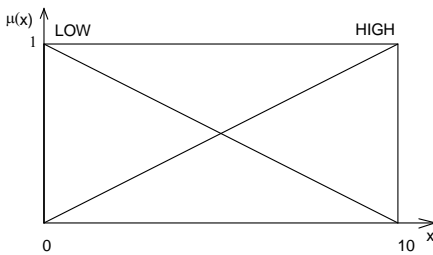


Fig. 5 Membership functions of input parameter- timetable influence

Fuzzy logic system comprises of 18 rules. Logical AND operator is employed with MIN (the rule of minimum for AND relationships). In creating the consequent fuzzy set, MAX – MIN inference is used. Defuzzification of the output fuzzy variable is by modified center of gravity method (COG) for emphasizing results of defuzzification of marginal aggregated output. Graph presentation of FPN subsystem for train delays is on Fig. 8.

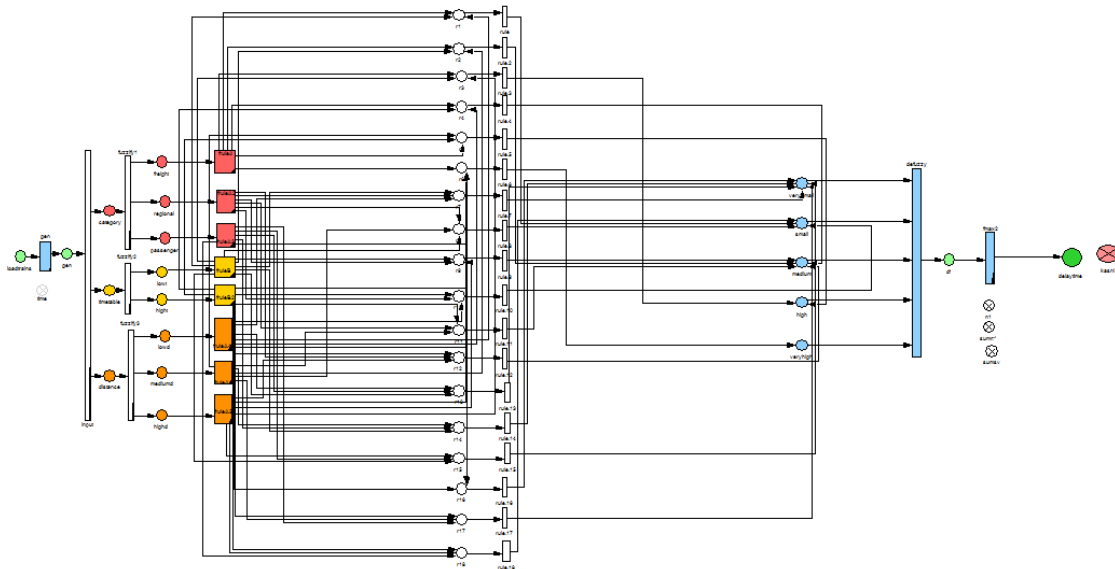


Fig. 8 FPN subsystem for train delays

## 6 Results and testing of model example

Example of train delay calculation is presented on Fig. 9 by fuzzy inference system of model. Test case is for following input parameters:

- train category – 5 (regional train);
- timetable influence – score 4;
- train distance traveled – 40 km;

Two rules are related to this set of input data: rule 7, and rule 20. Defuzzified output variable gives calculated train delay: 5.2 minutes.

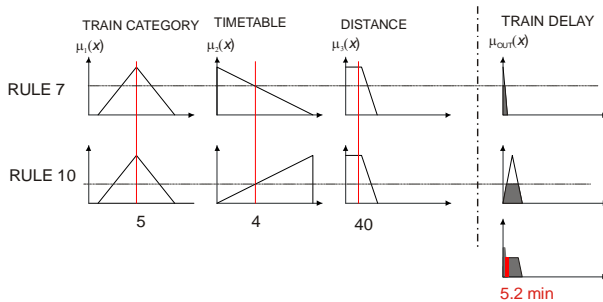


Fig. 9 Example of fuzzy inference system

During simulation, data are stored in external database. Latter, results can be classified and filtered for analysis. Sorting of data can be by section, by train number, by train relation, by train category. Multiple queries can be used to generate reports. Database can generate reports for comparing results of experimenting with changing timetables, section layout etc.

Two methods of generating trains were tested in the model: pulse timetable and deterministic timetable created by using official timetable. Timetable and train data are imported in model form external database.

Validation and verification of Petri Net model is by animation window in program and by simulation results analysis. Results from database are used to generate train time – distance diagram where any irregularities in model can be easily identified (Fig. 11).

Animation window is modeled to visually resemble to dispatcher interlocking control panel. Sections of model are animated by using section state data (Fig. 10).

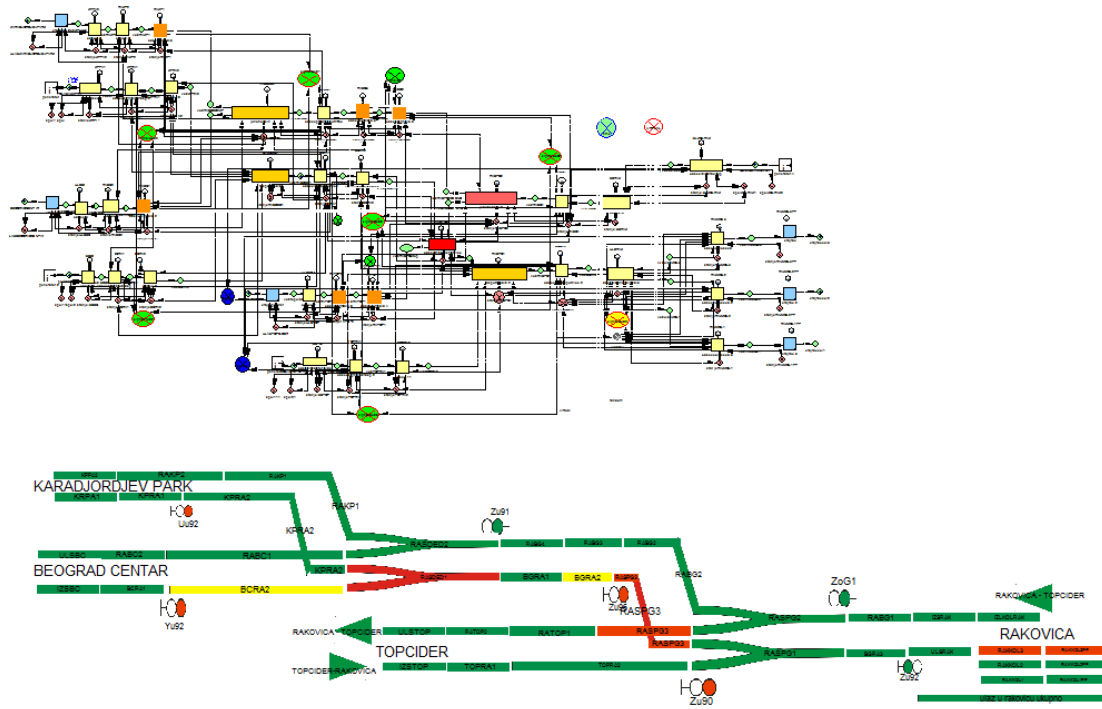


Fig. 10 PN graph with animation window

FPN model calculates train delay on entry in model. Propagation of these delays can cause knock-on delay on other trains. Simulation of trains traffic determines conflicts between train routes and identifies sections where secondary delays occur and trains involved.

Analysis of train time-distance graph generated by PN simulation results gives data on secondary delays. Fig. 11 presents a part of train time – distance diagram for sections of a model. Dashed lines are timetable train routes and continuous line (colors represent different

train categories) represent disturbed train routes with delay calculated in FPN. As presented in Fig. 11, train 124 has delay generated by FPN and cause disturbance on other train. Train cannot depart because of conflicting route with train 124. Train 125 is generating secondary delay on section TOPRA1 waiting for train 124 to release junction section.

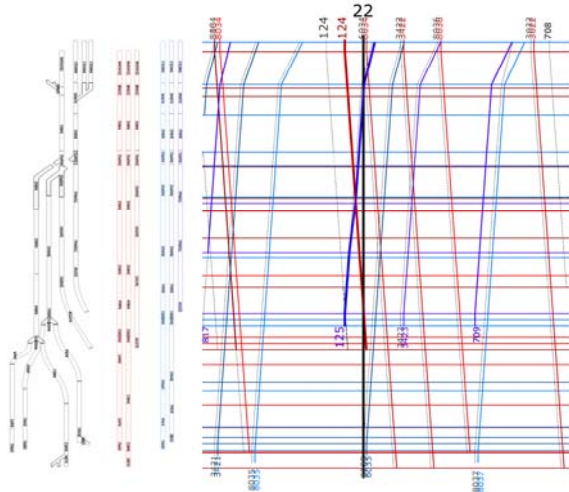


Fig. 11 Time - distance graph generated by simulation results

## 7 Conclusion

FPN can be used for modeling systems that are characterized by ambiguity and uncertainty. Train delays can be intuitively modeled. Knowledge of system behavior is in experience of experts who operates system (train dispatchers and operating staff). With expert knowledge and good train, timetable and infrastructure data, train delays in certain bounded system can be modeled by FPN. Since, FPN model is dependent on expert knowledge and timetable and infrastructure data, parameters of FPN should be defined for every specific case. That extends time of model implementation but enables more precise results. Testing FPN model and adjusting its input variables membership functions, rule base and output variable membership function can create Fuzzy Petri Net model for train delays that will produce quality results, comparable to real system delays.

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