THE FLOW MODELIZATION BY PETRI NET AND URBAN TRAFFIC

Marc Bourcerie, Frédéric Guegnard, Frédéric Bousseau

University of Angers LISA EA 4094,
4 Bd Lavoisier, 49000 Angers, France

marc.bourcerie@univ-angers.fr *(Marc Bourcerie)*

Abstract

The observation and the regulation of flows is very usefull in numerous domains of applications : in production systems, considering the flow of objects on a production line, in software ingeneering, considering data flow, in urbanism, considering the flow of vehicles. Among the applications of Petri Nets, there is a great number of examples in these domains in order to represent, to analyse the production systems and the conflicts management in urban traffic. Among others, these points need to solve some problems of resource sharing and complex sequencing. In this viewpoint, we propose now a modelization of regulation flow by Petri Net (PN). Then, we propose some models of flow measurement, in order to compare the occupation rate and some dynamical sequencers for a better dispaching of tasks in a real structure.
After a presentation of the principles on which the regulation takes place, we take up a specifical application : the flow regulation in an urban traffic. The exposed problem is relatively simple and could be extended to more complex situations.

Keywords: Petri Nets, flow control, urban network, modelization, real time, sequencers.

Presenting Author's biography

Marc Bourcerie obtained PhD degree in 1988. His work was about the aging of N-MOS transistor. In 1989, he joined the University of Angers, where he obtained "HDR" degree in 1996. In LISA laboratory, he works on the modelling of complex systems by generalized or coloured Petri nets. He is particularely interested by the modelization of flow and regulation in manufacturing, production systems by Petri Nets.

1 Introduction

In a numerous domains, it is necessary to notice the flows of objects. Control and surveillance of datas allow to bring a help to decision in regulation terms. So, upstream from a given point, we can make some changes which go to the sense of regulation. These are topical notions in numerous applications such as the flows of objects in production systems, surveillance of data flows or traffic in an urban network. Petri nets are very often used as tool of simulation and analysis. Their mathematical properties allows the presentation of complex systems and their behaviour analysis across the model.

So Petri nets are used in the field of production systems [14, 16, 18] and in study, management of flow for urban traffic [1, 10, 11, 12, 15, 17].

We use here this tool in the prospect of a help to decision. Presentation is supported with an example of surveillance of flow in an urban network which must be seen as the projection of various applications. It will be necessary therefore to enlarge the field of applications in domains related to.

We consider in a general way, a site constituted network of mono-directional arcs and intersection nodes. In this network, objects of identical natures (data or vehicles or components) circulate. Objective is to make sure that these objects are distributed at best in structure, according to some criteria to be defined and according to some constraints. We are therefore interested in a modular modelling of these problems with the help of PN, structure which is well adapted there.

In order to complete successfully this approach, we are going to use three types of structures:

-- The gauges of flow relating to 2 lines where circulate objects,

-- The gauges of occupancy rate of sites,

-- The sequencers able to drive objects toward two ways, in a given proportion, defined by the structure of these sequencers and answering to requirements.

By making dynamic the sequencer model we give the possibility of an interaction between gauges and sequencers. In fact, gauges are endowed with the capacity to provide a token when unbalance between flows or occupancy rates of sites, reach a predetermined threshold.

Having justified the well-grounded of these three types of structures, we are going to use the PN tool to lead to a modelling of the regulation of flows in a complex structure. With the help of the mathematical tool PN, the main points of the analysis of models will be approached, in objective to validate them.

2 Petri Nets: définitions

A Petri Net (PN) [8] is a quadruplet :

Q = <P, T, Pré, Post > such as :

P = {Pi}, i ∈{1,...,n} is called set of places.

T = {Tj}, j ∈{1,...,m} is called set of transitions with $P \cap Q = \varnothing$

Pre is an application of $P \times T \rightarrow N$

Post is an application of $P \times T \rightarrow N$

Pre (Pi,Tj) is called weight of the arc linking Pi et Tj.

Post (Pi,Tj) is called weight of the arc linking Tj et Pi.

Let be :

$W^- = \left[w^-_{ij} \right]$ where $w^-_{ij} = \mathrm{pre}(P_i, T_j)$

$W^+ = \left[w^+_{ij} \right]$ where $w^+_{ij} = \mathrm{post}(P_i, T_j)$

The incidence matrix is :

$$W = W^+ + W^- = \left[w_{ij} \right]$$

We call P-invariant a weighting of places $P^T = (\alpha_1, ..., \alpha_n)$ such as $P^T W = 0$, what means that $P^T \in KerW$ on left.

We call T-invariant a sequence $S = (\beta_1, ..., \beta_m)$ such as $WS = 0$. What means that $S \in KerW$ on right.

The evolution of markings follows the fundamental low :

$$M_f = M_0 + \Delta M = M_0 + WS \qquad (1)$$

where :

M_0 is the initial marking
W is the incidence matrix
S is the sequence of firing
M_f is the final marking obtained from initial marking after carrying out of sequence of firing.

These elements will be used in the following in order to validate the proposed models.

3 Measurement of flow : the gauges

The first element which we develop is relating to the picking up of information on the flows of tokens in models [4]. At first, we approach the principle of measure of absolute flow, then the principle of relating measurement of two flows.

3.1 Measurement of absolute flow

Preliminary operation consists in conceiving a model of picking up of flow of tokens on a line of PN (figure 1).

Here, we consider a line (whatever is application) represented by a succession of places and transitions linked up by arcs. The cell of measurement of flow (dotted lines) is constituted of a transition T1 which generates a token in the place P each time a token circulating on the line crosses this transition. We have then a counting of tokens on the line by simple counting of tokens in P. If we top up transition Tinit, validated by recurrent event *e*, the possibility of measurement of medium flow is possible.

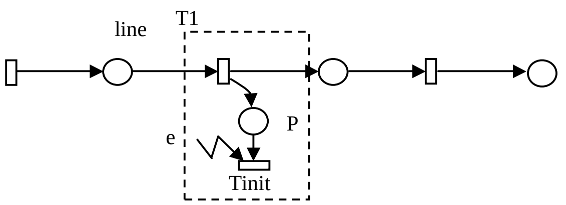

Fig.1 A cell for absolute flow measurement

3.2 Measurement of relative flow

It is very often deficient to operate on a measurement of absolute flow. The measurement of relating flow in real-time brings a strong information for the help to decision. The comparison of two or several flows on different lines of the site in real-time is going to allow upstream the adjustment of the flow of objects according to the requirements.

This measurement of relating flow is made thanks to various cells, more or less worked out. The basic cell of this type is introduced figure 2 (dotted lines).

The group represented by places P1, P2 and transitions T1, T, T2 is a sub-network constituting this basic cell. This network do not include P-invariant and has one T-invariant S = (1,1,1). His incidence matrix is the following :

$$\begin{array}{cc} & \begin{array}{ccc} T_1 & T & T_2 \end{array} \\ W = & \begin{pmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{array}{c} P_1 \\ P_2 \end{array} \end{array}$$

Final marking Mf is given from initial marking Mo and relation (1)§2.

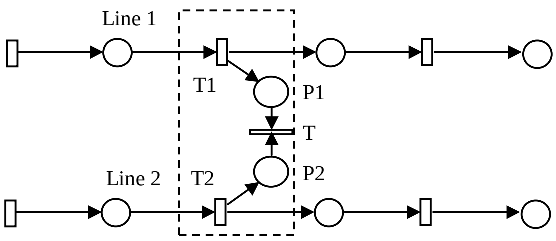

Fig.2 Basic Cell for relative flow measurement

Let be the firing sequence :

S = (n1, min(n1,n2), n2) meaning that the difference of flow on the two lines is n_1- n_2 if we have $n_1 > n_2$. Let be the initial marking $M_0 = (0,0)$.

The final marking is :

$$Mf = \Delta M = WS = \begin{pmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_2 \end{pmatrix} = \begin{pmatrix} n_1 - n_2 & 0 \end{pmatrix}$$

We have, in the place P1 (Resp P2), a number of tokens equal to the surplus of flow on the line 1 (Resp line 2) in comparison with the line 2 (Resp 1). The figure 3 represents a variant of this cell (in dotted lines). The possibility of watching the equilibrium of flows in real-time on both lines is now given. If both previous flows are equal, we have:

$$m(P_{12}) = m(P_{21}) = K \tag{2}$$

K having been chosen beforehand (here *K* = 3):

Incidence matrix of this net is :

$$\begin{array}{cc} & \begin{array}{cccc} T1 & T12 & T21 & T2 \end{array} \\ W = & \begin{pmatrix} +1 & -1 & 0 & 0 \\ 0 & +1 & -1 & 0 \\ 0 & -1 & +1 & 0 \\ 0 & 0 & -1 & +1 \end{pmatrix} \begin{array}{c} P1 \\ P12 \\ P21 \\ P2 \end{array} \end{array}$$

In this net is 1 P-invariant and 1T-invariant :

$$P^T = \begin{pmatrix} 0 & 1 & 1 & 0 \end{pmatrix} \qquad S = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$$

This P-invariant allows to write the equation of marking :

$$m(P_{12}) + m(P_{21}) = 2K \tag{3}$$

Let be now that n_1 tokens circulate on line 1 and n_2 tokens on line 2. Consequently, there is a difference of flow $0 \le n_1 - n_2 \le K$. The varying of marking after the measurement of relative flow is :

$$\Delta M = WS = \begin{pmatrix} +1 & -1 & 0 & 0 \\ 0 & +1 & -1 & 0 \\ 0 & -1 & +1 & 0 \\ 0 & 0 & -1 & +1 \end{pmatrix} \begin{pmatrix} n_1 \\ n_1 \\ n_2 \\ n_2 \end{pmatrix}$$

$$= \begin{pmatrix} 0, & n_1 - n_2, & n_2 - n_1, & 0 \end{pmatrix}$$

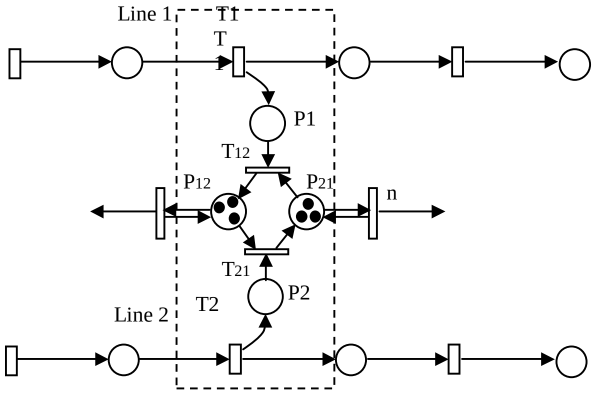

Fig.3 A cell for relative flow measurement

So, we obtain final marking where we can note the difference of flows, by reading of the marking of places P12 and P21:

$$M_f = M_o + \Delta M = \begin{pmatrix}0, & K, & K, & 0\end{pmatrix} + \Delta M$$
$$= \begin{pmatrix}0, & K + n_1 - n_2, & K + n_2 - n_1, & 0\end{pmatrix}$$

4 Occupancy rate

4.1 The model

The counting of number of objects on a site can be carried out by duplication of the place representing the site (figure 4a). A site (a node of network) can have several inputs (figure 4b) or several outputs.

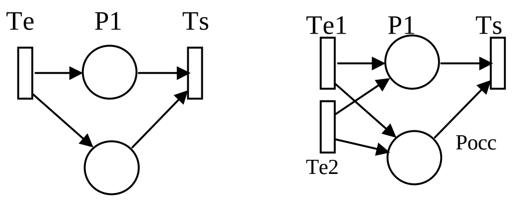

Fig 4a Site with one input 4b Site with two inputs

Let be $\varphi(e)$ the number of crossings of transition Te.

Let be $\varphi(s)$ the number of crossings of transition Ts.

We have, from an initial marking (0, 0) the occupancy rates which follow :

for the PN of figure 4a :

$$m(Pocc) = m(P_1) = \varphi(e) - \varphi(s)$$

for PN of figure 4b :

$$m(Pocc) = m(P_1) = \varphi(e_1) + \varphi(e_2) - \varphi(s)$$

It is however more useful to compare the occupancy rates of two sites. The figure 4c introduces two sites with two inputs and one output.

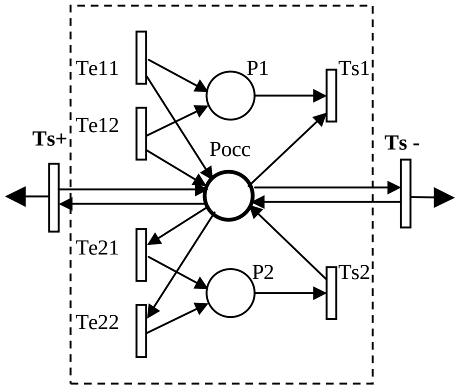

Fig.4c Comparison of occupancy rate

The place Pocc takes anti-tokens [6, 7], allowing a negative marking so. Both lateral transitions Ts + and Ts - provide a token as soon as the place Pocc contains “n” tokens or “n” anti-tokens (also called negative tokens).

So, we measure in real-time, the difference of the occupancy rates of places P1 and P2 and we perform an external order through transitions Ts + and Ts-. We are now going to study and validate this model across its incidence matrix and the using of the mathematical properties determined by invariants.

4.2 Study of model

PN of figure 4c (dotted lines) have 4 T-invariants (*S1* to *S4*) and 1 P-invariant. Let be *W* the incidence matrix.

$$W = \begin{array}{c} \begin{array}{cccccc} T_{11} & T_{12} & T_{S1} & T_{21} & T_{22} & T_{S2} \end{array} \\ \begin{pmatrix} +1 & +1 & -1 & 0 & 0 & 0 \\ +1 & +1 & -1 & -1 & -1 & +1 \\ 0 & 0 & 0 & +1 & +1 & -1 \end{pmatrix} \end{array} \begin{array}{c} P_1 \\ P_{occ} \\ P_2 \end{array}$$

Invariants are :

$$S_1 = \begin{pmatrix}1 & 0 & 1 & 0 & 0 & 0\end{pmatrix}$$
$$S_2 = \begin{pmatrix}0 & 1 & 1 & 0 & 0 & 0\end{pmatrix}$$
$$S_3 = \begin{pmatrix}0 & 0 & 0 & 1 & 0 & 1\end{pmatrix}$$
$$S_4 = \begin{pmatrix}0 & 0 & 0 & 0 & 1 & 1\end{pmatrix}$$

$$P^T = \begin{pmatrix}1 & -1 & -1\end{pmatrix}$$

From this P-invariant and initial marking (0, 0, 0) we have the equation of marking :

$m(P_1) - m(P_{occ}) - m(P_2) = 0$, what means that:

$$m(P_{occ}) = m(P_1) - m(P_2) \qquad (4)$$

For the model of figure 4c, we obtain the differential measurement of occupancy rate :

$$m(P_{occ}) = \sum_{i=1}^{2} \varphi(e_{1i}) - \sum_{i=1}^{2} \varphi(e_{2i}) - \varphi(s_1) + \varphi(s_2)$$

Then, this model allows the measurement of the difference of marking between places P1 and P2, nevertheless without disturbing of circulation of tokens on both lines.

Previously [3,5], some models of sequencers and dynamic sequencers by PN and by coloured PN were introduced for a modelization of sites of production.

We introduce now a model of static sequencer and its transformation to dynamic sequencer, well adapted to the example which is going to be approached then.

5 Control of flow : The sequencer model

5.1 Static model

The model presented figure 5a allows to distribute 2/5 tokens on the line 1 and 3/5 tokens towards the line 2. This model, introduced as an example of distribution, will be later exploited to represent the dispatching of objects from the point (2,1) of figure 8 towards points (2,2) or (3,1).

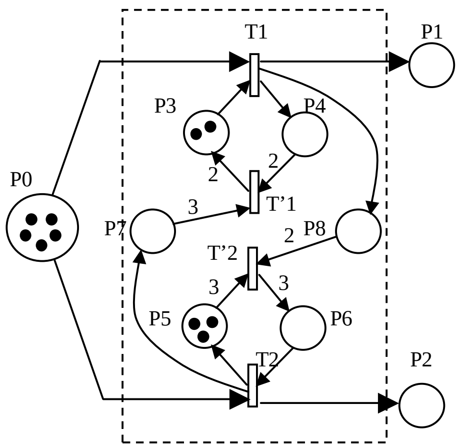

Fig.5a Static sequencer

5.2 Model analysis

The Petri net of figure 5a has 5 P-invariants and not any T-invariant. The incidence matrix W of this net, the 5 P-invariants and initial marking M0 are:

$$W = \begin{array}{c} \begin{array}{cccc} T1 & T'1 & T'2 & T2 \end{array} \\ \begin{pmatrix} -1 & 0 & 0 & +1 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 \\ -1 & +2 & 0 & 0 \\ +1 & -2 & 0 & 0 \\ 0 & 0 & -3 & +1 \\ 0 & 0 & +3 & -1 \\ 0 & -3 & 0 & +1 \\ +1 & 0 & -2 & 0 \end{pmatrix} \end{array} \begin{array}{c} \\ P0 \\ P1 \\ P2 \\ P3 \\ P4 \\ P5 \\ P6 \\ P7 \\ P8 \end{array}$$

$$P_1^T = (0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0)$$
$$P_2^T = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0)$$
$$P_3^T = (0 \quad 0 \quad 0 \quad 3 \quad 0 \quad 0 \quad 2 \quad 2 \quad 3)$$
$$P_4^T = (1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$$
$$P_5^T = (0 \quad -1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2 \quad 3)$$

$$M_0 = (5 \quad 0 \quad 0 \quad 2 \quad 0 \quad 3 \quad 0 \quad 0 \quad 0)$$

A possible firing sequence from this initial marking is :

$$S = (2 \quad 1 \quad 1 \quad 3)$$

Then we obtain the variation of marking :

$$\Delta M = (-5 \quad 2 \quad 3 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$$

And the final marking, using the formula (1)§2 :

$$M_f = M_0 + \Delta M = (0 \quad 2 \quad 3 \quad 2 \quad 0 \quad 3 \quad 0 \quad 0 \quad 0)$$

This result confirms definitely that 2/5 of the tokens were distributed towards the line 1 and 3/5 towards the line 2. The PN delimited by the dotted line has 1 T-invariant and 3 P-invariants:

$$S = (2 \quad 1 \quad 1 \quad 3)$$

$$P_1^T = (1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0)$$
$$P_2^T = (0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0)$$
$$P_3^T = (3 \quad 0 \quad 0 \quad 2 \quad 2 \quad 3)$$

Notice that the previous sequence S is a T-invariant of this sub-net. Then, this sequence S brings back therefore this PN to its initial marking, allowing so a new distribution of tokens arriving in the place P0 according to the same ratios 2/5, 3/5

5.3 Dynamical model

The model of the figure 5b is a variant of the previous one. The objective is here to obtain a dynamic model making vary the proportions of tokens aimed to the one or the other line in case of disturbances applied to network. We operate by taking in the source (place P0) a given number of tokens (grey transitions having priority). These tokens are not therefore counted by the sequencer.

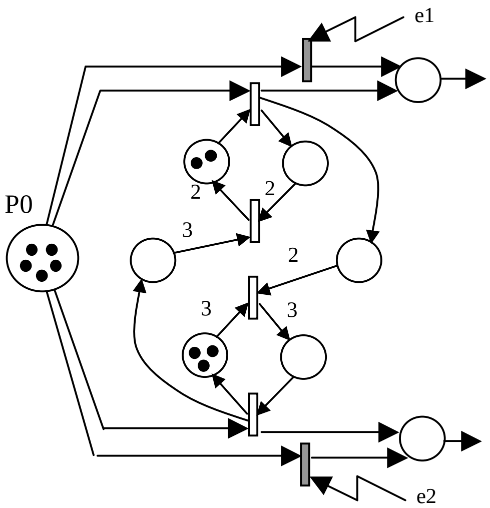

Fig.5b Dynamical sequencer

6 The urban traffic

As an example, we are going to introduce a model of urban network with mono-directional circulation. This picture (figure 6) comes in form of rectangular network with dimensions (h,v) (h for horizontal and v for vertical). The objects move from the point A to the point B without step backwards (in dotted line, a possible way).

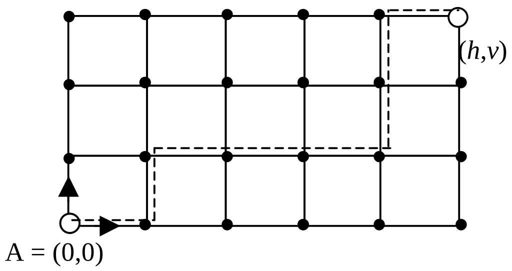

Fig.6 . An urban traffic net model

Classically, it is known that the number of possible ways for this problem is :

$$N = \binom{v+h}{h} = \binom{v+h}{v} = \frac{(v+h)!}{v!\,h!}$$

At first, we assume that after every node, objects make for both following nodes (two neighbouring nodes) with an equal probability. It follows that the density of tokens and probability of blockage is not uniform for all the nodes of the network. So, the figure 7a represents this rate normalized by occupation of nodes, by taking norm 1 for points A and B. The figure 7b introduces the result to be acquired, if we want to have an equilibrium of the load on every node.

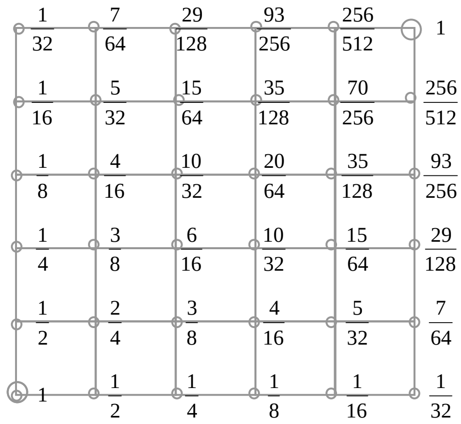

Fig.7a Occupancy rate for an equiprobable way

For every node, in order to overcome this disadvantage, and to obtain the result presented figure 7b, it is necessary to arrange unequally the objects to the neighbouring nodes.

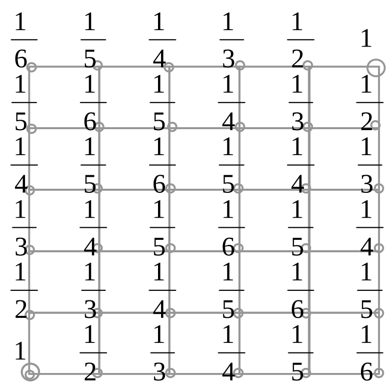

Fig 7.b Corrected ways

The coefficients associated to each links between two nodes are specified figure 8. These coefficients allow us to get the results of figure 7b.

This example shows that flows on two directions are not necessarily equal if objective is to balance the occupancy rate of sites.

In order to create the models representing these problems, it is therefore necessary to use workable sequencers such as those who were presented in §5.

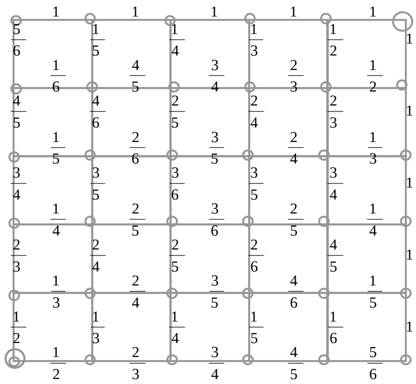

Fig.8 Coefficients on the links

7 Regulation of flows

7.1 Regulation loop of flows

We are intentionally situated at the level of the model PN to develop the principle of regulation of flows, considering the modelling of two "rival" lines (two possible ways from a node). The surveillance of the instantaneous flow of tokens is possible on each of these two lines. The comparison of these flows by a gauge allows us to act upstream through a sequencer and obtain a regulation of flows (figure 9a).

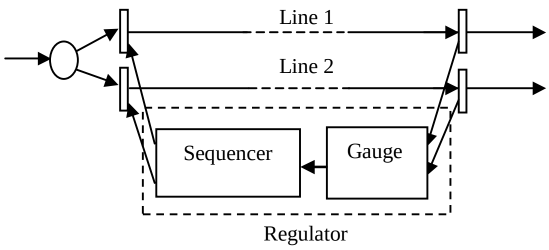

Fig 9a. Regulator with gauge of flow

The regulator, constituted by the gauge and the appropriate sequencer, allows to take a sample of the information of flows, to compare them by the gauge and to provide a sequence of distribution of objects by the upstream sequencer.

This schema can be adapted to the measurement of difference of occupancy rate at the level of two places of both lines (figure 9b).

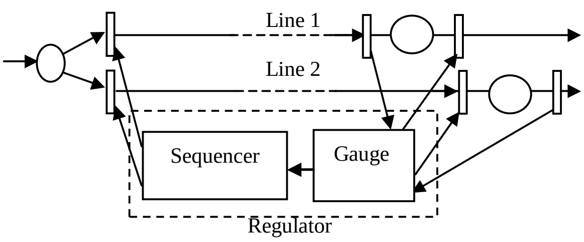

Fig 9b. Regulator with gauge of occupancy rate

This last schema leads to the first presentation of a system of regulation modelled by a Petri net. (figure 10). This model is worked out by using the building of figures 4c and 5b, relating to a gauge of measurement of occupancy rate and to a dynamic sequencer.

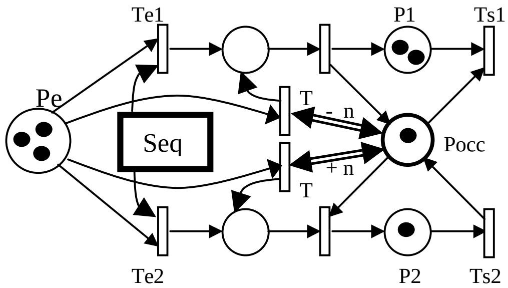

Fig 10. Regulator by Petri net

As soon as the difference of occupancy rate on places P1 and P2 exceeds “n” tokens in absolute value, one of both transitions T injects a token on one or other line.
The initial marking of this example points out that :

Three objects are going to be directed to one of two lines, managed by the sequencer (Place Pe).

Two objects circulate on the line 1(Place P1).

One object circulates on the line 2(Place P2).

Contents of the place Pocc points out that there is a difference of one object in term of occupancy rate of both lines, to advantage of the line 1.

When this difference will have reached the value "n", a token will be injected on the line 2. This token will not be taken into account by the sequencer.

7.2 Autonomous functioning of regulator

In order to correctly analyse the functioning of the regulator, it is necessary to observe it in autonomous functioning :

Transitions Ts1, Ts2, Te1 and Te2 are closed.

There is enough tokens in input place Pe (in order to validate the transitions T).

There are bi-directional arcs between place Pocc and transitions T. Then, the firing of T does not alter the marking of Pocc. This part of the model cannot be taken into account by incidence matrix. Consequently, a mathematical analysis of these two links is not possible.

Let be the hypothesis that the difference of occupancy rate is “k” tokens (with k ≥ n) to advantage of the one or the other line.

There are therefore two initial possible conditions of marking on the three places P1, Pocc, P2:

1st case:

The marking (*m(P1), m(Pocc), m(P2)*) follows the states :

$$(k, \quad k, \quad 0) \rightarrow (k, \quad k-1, \quad 1) \rightarrow ....$$
$$.... \rightarrow (k, \quad n-1, \quad k-n+1)$$

$\forall k \geq n$.

Because k ≥ n, the transition T of line 2 is fired, injecting a token on line 2. Consequently, the marking of place P2 increases and the marking of place Pocc decreases, as long as k ≥ n .

In the particular case with $n=1$, the final marking is :

$Mf = (k \quad 0 \quad k)$, leading to an equilibrium among tokens on every line.

2nd case:

The marking (*m(P1), m(Pocc), m(P2)*) follows the states:

$$(0, \quad -k, \quad k) \rightarrow (1, \quad -k+1, \quad k) \rightarrow ...$$
$$... \rightarrow (k-n+1, \quad -n+1, \quad k)$$

$\forall k \geq n$

If $n=1$, the final marking is :

$Mf = (k \quad 0 \quad k)$, this also leads to an equilibrium among tokens on every line.

Notice that, in this last case, we must consider negative tokens (or anti tokens) [8, 9, 13].

7.3 Application to the traffic regulation

In the previous paragraphs, we introduced three kinds of models:

-- *The gauges of flow*; taking a sample of information to compare the flows.

-- *The gauges of occupancy rate*; taking a sample of information to compare the occupancy rate of sites.

-- *The sequencers*; distributing objects from a site towards several other sites. The dynamic sequencers are able to change in real-time the protocols of sharing out of objects towards these sites.

Interconnections of these three kinds of models are dependent on requirements and every site has a specific shape. It seems therefore a priori difficult to draw a methodology of standardized interconnection. However, it appears that criteria of measurement of flow and occupancy rate are jointly to take into account for a better sharing out of objects by way of the dynamic sequencers.

A weak flow of objects does not mean a non-saturation of sites therefore (see traffic jams): for a perfectly free way and a completely saturated way, the measurement of flow is the same: the measurement of flow is not therefore enough.

The measurement of occupancy rate of sites is then more significant, although not implicating saturation of paths necessarily: this measurement can therefore be deficient.

The action on a given structure can moreover be considered at several levels:

Either at local level, by acting on the feeding of the closest sites, or at more general level where it can be possible to act more upstream of network to orientate objects.
We take back the example of the urban network of the figure 8 by introducing a structure of regulation of flow at the level of the single "cell". In other words, the action is developped on the closest nodes of the network.
We apply results obtained before, notably by using of the model presented figure 10. On the figure 11, two sub-structures are used:

-- The comparative measurement of occupancy rate of two sites with 2 inputs and 1 output

-- The dynamic sequencer S.

The place Q (x, y) (eventually with anti-tokens) measures the difference of occupancy rate of places P(x, y+1) and P(x+1, y) according to the principle presented figure 4c.

Transitions T are ordered by the sequencer S, made responsible for establishing the common sequence, as it is presented figure 5a.

Grey transitions have priority and allow to put off tokens out of the control of the sequencer, making it dynamic (figure 5b).

Let be m(Q) the number of tokens in the place Q. One of these two grey transitions is fired if:

$m(Q) \geq |n|$ where $n \in Z$ (figure 4c and 10).

This situation of rocking of occupancy rates will occur under the influence of disturbances:

-- by an external action: supplying of tokens or catching of tokens at the level of places P(x, y+1) and P(x+1, y) (these disturbances are not represented on these models).

-- by an internal cause, in case of slowing down of objects (therefore of tokens in the cell). The introduced delay times are not represented in these models.
This means that the objective of this model is to represent the regulation, by considering possible disturbances without representing them therefore (what would be wholly accomplishable, but that not presented for reasons of readability).

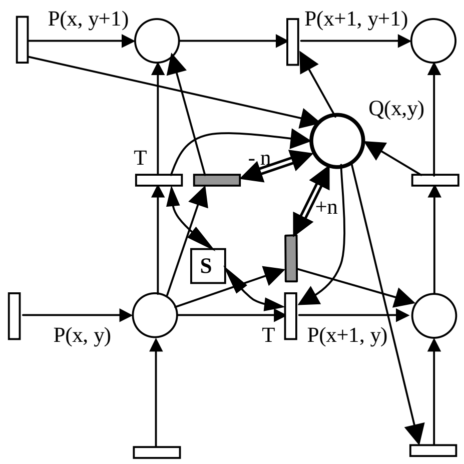

Fig.11. Regulation for imediate proximity

8 Conclusion

We reviewed the various models constituting the whole regulation of flow applied to a problem of urban network.

It appears that Petri Nets allows to represent and to validate regulation systems. The basical models which have been presented here can then be put together in a modular way. Mathematics of PN allow the validation of models and the functioning of these models can be simulated with the help of devoted software.

Lastly, we introduced the model of a regulator of flow in immediate nearness. This model can be duplicated to envisage an action on several cells with a regulation in nearness. We can also study a model of regulation on a more large scale.

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