EVIDENCE-BASED MATHEMATICAL MAINTENANCE MODEL FOR MEDICAL EQUIPMENT

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Abstract

Measuring the availability of medical equipment based on various maintenance types has been a major concern for hospitals. Most of methodologies used were either theoretical proposals or empirical and very little has been done using mathematical modelling. A mathematical model is developed using a mixed integer based approach for maintenance operations schedules for medical equipment. A preliminary proposed model to analyze the probability of failures was developed and used to analyze data from hospitals in South Africa and the United States of America. A greedy algorithm was used to measure availability of anaesthetic machines using maintenance data available and a promising results regarding optimisation of maintenance schedules was found using simulations on Matlab. More data on maintenance of other equipment is being collected and the mathematical model that was developed using a mixed integer approach will be tested.

Keywords: maintenance, mixed integer, availability, reliability, failures.

Presenting Author's biography

Abdelbaset Khalaf is a senior lecturer in clinical engineering at Tshwane University of Technology (TUT). He earned his first degree B.Sc. in Biomedical Engineering from METU in Turkey in 1984 and MTech in Clinical Engineering from TUT in 2004. Abdelbaset is a consultant in Healthcare technology Management and advisor to the World Health Organisation (WHO) and department of health in South Africa and the Sub-Saharan African Region.

1 Introduction

A serious debate about preventive maintenance (PM) intervals is taking place among clinical engineering practitioners on various levels and in professional journals. The debate is focused on the standard requirements by regulating authorities that OM intervals should follow the equipment manufacturer's recommendations.

Some devices that appear to be very similar in their function and design have manufacturer-recommended intervals that vary by a factor of two or more [1]. The question would have been raise about the credibility of these recommended intervals and whether it is based on meaningful test data. Experience showed that equipment manufacturers are reluctant to share that information if there are any documented data.

Judging maintenance outcomes based on preventive maintenance (PM) or safety and performance inspection (SPI) is not possible and the same applies to periodic replacement of parts or calibrations. Therefore, we need to measure outcomes such as uptimes and failure rates as part of our PM and possibly look at different strategies for corrective maintenance. By doing that we may have an answer to the question of how to prove that our maintenance program is both effective and efficient [2].

In answering the question; why Evidence-based maintenance (EBMain) for medical equipment we should admit that no medical device is 100% safe and no one has unlimited resources. Thus we need to find the best balance between our needs and resources, being effective and efficient. [2]

Literature review has shown that very little has been done to measure availability of medical equipment in relation to maintenance using mathematical modelling and the empirical approaches is not widely used where other sectors of industry have developed various mathematical models to measure availability and reliability of equipment and systems.

According to John Endrenyi and George Andres, maintenance policies based on mathematical models are much more flexible than heuristic policies and the great advantage of the mathematical approach is that the outcomes can be optimised and maximum reliability or minimal cost can be achieved [3].

2 Modelling the probability of failure of equipment

The probability of the equipment being operational is considered to have the form of an exponential function which depends on both, the age of the equipment and the date of last maintenance. Consider the following function:

$$P_{op}(t) = P_{op}(t_0)\, e^{-bt} e^{-a(t-r)} \tag{1}$$

where

- $P_{op}(t)$ is the probability of the unite being operational at time t,
- a is the parameter related to maintenance,
- b is the equipment aging parameter and
- r is the date of last repair or maintenance.

Since the representation of this term for the maintenance will be made in discrete time, the next step is to represent the probability of the unit being operational in a recursive manner as a function of the time period. Consider the time periods of duration h, and then if no maintenance is made at time t, this may be represented as

$$d(t+h) = \alpha\, d(t)$$

where

$$\alpha = e^{-ah} e^{-bh} = e^{-(a+b)h}$$

If the unit is maintained at time t then

$$d(t+h) = \beta\, d(t)$$

where

$$\beta = e^{-bh}$$

Preliminary maintenance data on anaesthetic machine was collected from different hospitals and used to calculate α and β. Table 1 gives examples of collected maintenance data for a given class of equipment (Anaesthetic machines). From this data, we extract maintenance data for 9 units. We compute the average cumulative number of failures for Anaesthetic machines at each period. The results are reported Figure 1 in red circles. In order to compute the parameters of function (1), integral form of this function is made to approximate the average cumulative number of failures. Nonlinear fitting and results are reported in figure 1 in dashed blue.

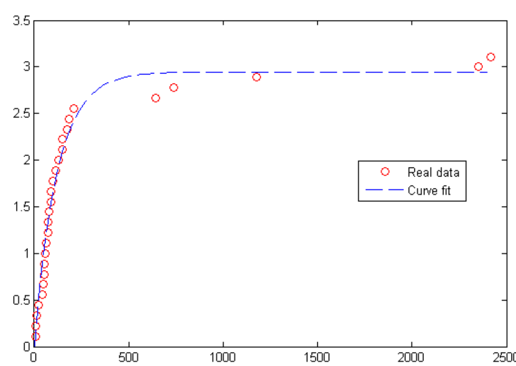

Fig 1: Probability of failure approximation

Tab 1: Example of maintenance data (Source Hospital in South Africa)

Aneasthetic machines

Unit	Date of Purchase	Date of Maint.	Date of Repair	Unit	Date of Purchase	Date of Maint.	Date of Repair
1	01/01/04	15/05/07	23/03/07	6	01/08/02	31/05/05	07/03/07
		20/11/07	23/03/07			21/11/07	06/06/07
			12/08/07				03/06/08
		28/05/08				28/05/08	03/06/08
		28/11/08				28/11/08	23/06/09
			23/06/09			01/05/09	
		01/05/09	15/07/09			01/11/09	
		01/11/09	28/09/09	7	01/09/00	15/05/07	10/10/07
2	10/08/01	29/05/07				20/11/07	22/10/09
		20/11/07	04/03/09				01/12/07
		28/05/08				28/05/08	14/12/07
		28/11/08				28/11/08	25/06/09
		01/05/09					04/04/09
		01/11/09				01/05/09	
3	10/08/01	24/05/07				01/11/09	
		20/11/07		8	05/05/03	05/06/07	14/01/09
		28/05/08				21/11/07	12/02/09
		28/11/08					24/08/09
		01/05/09				28/05/08	
		01/11/09				28/11/08	
4	01/08/02	06/06/07	25/02/09			01/05/09	
		20/11/07	30/06/09			01/11/09	
		28/05/08		9	31/10/06	24/05/07	24/07/07
		28/11/08				20/11/07	03/08/07
		01/05/09				28/05/08	12/01/09
		01/11/09				28/11/08	
5	01/09/00	24/05/07	08/02/07			01/05/09	
		20/11/07	15/04/07			01/11/09	
			30/05/09				
		28/05/08					
		28/11/08					
		01/05/09					
		01/11/09					

3 Mixed integer approach for maintenance scheduling. [4, 5]

The objectives of our research concern mainly a development of new evidence based mathematical models and methods dedicated to model based diagnosis and maintenance for medical equipments. This modelling approach will be based on extensive dedicated collected data from state hospitals, private hospitals groups and suppliers of medical equipment.

Several approaches have been proposed in order to manage equipment maintenance scheduling taking into account several parameters like equipment aging, equipment availability, risk categories and mission criticality.

The complexity of medical equipment maintenance problem is based on decision based information variables that are difficult to handle. The proposed model is a preliminary idea in order to come up with a systematic optimization approach that handles variables of different types of binary integers and real variables. A mixed integer approach is proposed for equipment maintenance scheduling optimization.

This approach focuses on the level of availability of medical equipment for healthcare service delivery. Depending on the mission, the healthcare provider can choose a minimum set of equipment of same type that must be available at any time and accordingly plan the scheduled maintenance.

3.1 Problem formulation

Let K be the number of units of the same type.

Let $x_k(t)$ the binary maintenance variable considered at time t for the equipment $k \in K$, given by:

$$x_k(t) = \begin{cases} 1 & \text{if k is undermaintenane} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Let $d_k(t)$ be the availability information or function of equipment k, evaluated at time t. This function is a monotically decreasing function and mainly depends on usage and maintenance provided to the equipment.

With no maintenance provided to equipment k

$$d_k(t+1) = \alpha_k d_k(t) \quad (3)$$

With maintenance provided to equipment k

$$d_k(t+1) = \beta_k [1 - d_k(t)] + d_k(t) \quad (4)$$

This leads to:

$$d_k(t+1) = \begin{cases} \alpha_k d_k(t) & \text{if } x_k(t) = 0 \\ \beta_k [1 - d_k(t)] + d_k(t) & \text{otherwise} \end{cases}$$

This equation may be re-written as

$$d_k(t+1) = \beta_k x_k(t) + \alpha_k d_k(t) + (1 - \beta_k - \alpha_k) d_k(t) x_k(t)$$

(5)

Let $z_k(t) = d_k(t)x_k(t)$

Then:

$$d_k(t+1) = \beta_k x_k(t) + \alpha_k d_k(t) + (1-\beta_k-\alpha_k)z_k(t) \quad (6)$$

$z_k(t)$ must satisfy the following constraints:

$$0 \le z_k(t) \ge x_k(t)$$

$$d_k(t) + x_k(t) - 1 \le z_k(t) \ge d_k(t)$$

Special case: $\beta_k = 1$ then

$$d_k(t+1) = x_k(t) + \alpha_k d_k(t) - \alpha_k z_k(t) \quad (7)$$

With $\gamma_k = 1 - \beta_k - \alpha_k, k \in 1,\ldots,K$

Let:

$$A = \begin{bmatrix} \alpha_1 & & \\ & \ddots & \\ & & \alpha_k \end{bmatrix};\ B = \begin{bmatrix} \beta_1 & & \\ & \ddots & \\ & & \beta_k \end{bmatrix};\ D = \begin{bmatrix} \gamma_1 & & \\ & \ddots & \\ & & \gamma_k \end{bmatrix}$$

This leads to:

$$d_k(t+1) = Ad_k(t) + Dz_k(t) + Bx_k(t) \quad (8)$$

Our objective is to compute the optimal schedule for maintenance. Based on our formulation, any Mixed Integer software can be used. However, due to the complexity, an initialisation procedure is needed. In this paper a greedy algorithm is proposed to solve this problem.

3.2 Greedy algorithm

The proposed greedy algorithm is given below.

Input: $\alpha_k, \beta_k, d_k(0), \forall k \in K$
Threshold availability for units
Minimum number of units available

Output: $x_k(t), \forall k \in K, t = 1 \cdots T$

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Begin Algorithm
for All Periods t
    for All units k
       Calculate dk(t) = αk dk(t−1), ∀k ∈ K
        if dk(t) < threshold(k)
             xk(t)=1
             dk(t) = βk ;
        end if
    end for
    Sort dk(t) for all units
      in ascending order;
      for All units k
          s = Σ dk(t)
        if s < minimum availability
                of units
            ko=Ascending order(k)
            xko(t)=1
            dko(t) = βk ;
        end if
      end for
end for

End Algorithm
```

3.3 Simulation results

Simulations were done on Matlab for the greedy algorithm. The hypotheses for simulation are as follows:

- Number of units: 9
- Minimum number of units to be available: 3
- Minimum availability for each unit: 0.2
- Purchase date: All units were supposed to be purchased at the same date

In table 2 below, the results obtained by the greedy algorithm are given. The results show better maintenance scheduling.

4 Conclusion

In this paper, a mixed integer model for the programming of maintenance for biomedical equipment was presented. The parameters of the model were identified from data on equipment collected from South African Hospitals. The model was solved using a greedy algorithm. This will be integrated within a solver using a branch and bound algorithm.

Tab2: Example of maintenance data

Aneasthetic machines					
Unit	Date of Purchase	Date of Maint.	Unit	Date of Purchase	Date of Maint.
1	01/01/08	01/03/08	6	01/01/08	01/06/08
		01/09/08			01/01/09
		01/04/09			01/08/09
		01/11/09			01/03/10
		01/06/10			01/10/10
			7	01/01/08	01/06/08
2	01/01/08	01/03/08			01/01/09
		01/10/08			01/08/09
		01/05/09			01/03/10
		01/12/09			01/10/10
		01/07/10			
3	01/01/08	01/04/08			
		01/11/08	8	01/01/08	01/06/08
		01/06/09			01/01/09
		01/01/10			01/08/09
		01/08/10			01/03/10
					01/10/10
4	01/01/08	01/05/08			
		01/12/08			
		01/07/09	9	01/01/08	01/06/08
		01/02/10			01/01/09
		01/09/10			01/08/09
					01/03/10
5	01/01/08	01/06/08			01/10/10
		01/01/09			
		01/08/09			
		01/03/10			
		01/10/10			

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