

FUZZY-BASED MODELLING OF AN MR DAMPER

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Abstract

A Magneto-Rheological (*MR*) damper exhibits a hysteretic and non-linear behavior. This behavior makes it a challenge to develop a model for this system. The present research is centered on proposing and analyzing two different fuzzy models of an *MR* damper based on experimental data. The first model uses an Adaptive Neuro-Fuzzy Inference System (*ANFIS*) and the second combines fuzzy methods with semi-phenomenological models. The results showed that fuzzy modelling can be a powerful framework to capture the behavior of highly non-linear systems. Among the various input patterns analyzed, stepped electric current signals allowed a better training of the *ANFIS* model. Both proposed structures obtained Error to Signal Ratio (*ESR*) values of less than 0.1 for the majority of the experiments. This intensive experimental study confirmed previous theoretic work done for *MR* damper model fitting.

Keywords: MR Damper, Modelling, Fuzzy, ANFIS, Non-Linear Systems.

Presenting Author's Biography

Ruben Morales-Menendez holds a PhD Degree in Artificial Intelligence from Tecnológico de Monterrey. From 2000 to 2003, he was a visiting scholar with the Laboratory of Computational Intelligence at the University of British Columbia, Canada. For more than 23 years, he has been a consultant specializing in the analysis and design of automatic control systems for continuous processes. He is a member of the National Researchers System of Mexico (Level I) and a member of IFAC TC 9.3.



1 Introduction

Magneto-Rheological (*MR*) fluids are non-colloidal suspensions of particles with a size on the order of a few microns [1]. These fluids are unique due to their ability to change their properties reversibly between fluid and solid-like states in milliseconds upon the application of a magnetic field.

Among a broad spectrum of applications, *MR* fluids have been widely utilized for vibration damping systems. In the past decade, there has been an increasing interest of scientists and engineers on these *MR* fluid dampers and their applications [2]. *MR* fluids are appealing for damping systems since they can operate at temperatures ranging from 40 to 150 °C with only slight variations in the yield stress. Additionally, *MR* fluids are almost insensitive to impurities and can be controlled with low voltages (12-24 V) and an electric current driven power supply outputting 1-2 A [3].

An *MR* damper can be regarded as a semi-active suspension system. These systems offer the reliability of passive devices, but maintain the versatility and adaptability of active systems. A semi-active *MR* damper is a non-linear dynamical system, where the inputs can be the elongation speed and the electric current. The current is the control input that modulates the damping characteristic of the *MR* fluid through the variation of a magnetic field. The output is the force delivered by the damper.

Although *MR* dampers are greatly promising for control scenarios, their major drawback lies on their non-linear and hysteretic behavior, Fig. 1. Furthermore, the design of a controller generally requires to model the system, which becomes a non-trivial task when it comes to *MR* dampers [4].

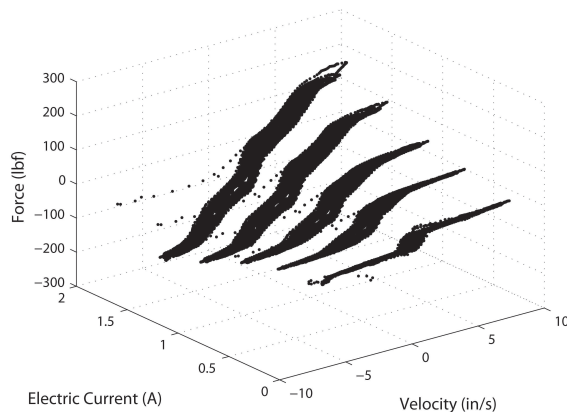


Fig. 1 *MR* damper behavior. Force is plotted against velocity at five electric current inputs.

In [5], an Adaptive Neuro-Fuzzy Inference System (*ANFIS*) was explored for fitting simulated data obtained with the Spencer model, [2]. The *ANFIS* was proven to fit the simulated data to an acceptable degree. Similar results can be observed in [6] and [7].

The present study is motivated on the aforementioned challenge that involves the correct modelling of an *MR* damping system. Two fuzzy models are proposed and analyzed. Experimental data sets were obtained at the mechanical testing laboratory of Metalsa¹. Section 2 presents a literature review. Section 3 discusses the experimental setup and design of experiments (*DoE*). Sections 4 and 5 describe the models proposed and the results. Section 6 discusses the results. Finally, section 7 concludes the research.

Tab 1 defines the variables that will be used through the paper.

Tab. 1 Variables.

Variable	Description
$x(t)$	Linear displacement, in
$v(t)$	Linear velocity, in/s
$i(t)$	Electric current, A
$F(t)$	<i>MR</i> Damper output force, lbf
$\hat{F}(t)$	Estimated <i>MR</i> damper force, lbf
T	Total number of discrete samples

2 Literature Review

2.1 MR Damper Modelling

Different modelling techniques have been studied for *MR* dampers. In [2], [8], and [4] phenomenological modelling techniques have been explored in order to obtain *MR* damper models. In [9], semi-phenomenological techniques were used to develop a mathematical model able to describe the hysteretic behavior of the *MR* damper. The research done in [3] compared three different model structures for the *MR* damper including a black-box one based on Non-linear *ARX* (*NARX*) models. The performance index selected by the author for comparing the results was the Error to Signal Ratio (*ESR*). The index value is one if the model is trivial and zero if the model is perfect. The definition for the *ESR* is shown in Eq. 1 taken from [3], where T indicates the total number of discrete samples.

$$ESR = \frac{\frac{1}{T} \sum_{t=1}^T (F(t) - \hat{F}(t))}{\frac{1}{T} \sum_{t=1}^T \left(F(t) - \left(\frac{1}{T} \sum_{j=1}^T F(j) \right) \right)^2} \quad (1)$$

2.2 Fuzzy Modelling

A Takagi-Sugeno-Kang (*TSK*) fuzzy model can be selected for modelling complex systems. The fuzzy rules of the model can be determined by adaptively generating them based on input and output data or by selecting them by hand. The inputs of the model are fuzzy and the outputs are crisp. The total output of the system is calculated using the weighted average of the output functions [6]. The system can use a hybrid learn-

ing algorithm that combines the backpropagation gradient descent and least squares methods. A *TSK* fuzzy model trained in this manner is often named Adaptive Neuro-Fuzzy Inference System (*ANFIS*).

If a first-order *ANFIS* consists of three inputs and one output (each input with three possible membership functions), and only three fuzzy rules are selected as shown in Eqs. 2 - 4,

$$\begin{aligned} &\text{If } x(t) \text{ is } A_1 \text{ and } v(t) \text{ is } B_1 \text{ and } i(t) \text{ is } C_1 \\ &\text{then } f_1(t) = p_1x(t) + q_1v(t) + r_1i(t) + s_1 \end{aligned} \quad (2)$$

$$\begin{aligned} &\text{If } x(t) \text{ is } A_2 \text{ and } v(t) \text{ is } B_2 \text{ and } i(t) \text{ is } C_2 \\ &\text{then } f_2(t) = p_2x(t) + q_2v(t) + r_2i(t) + s_2 \end{aligned} \quad (3)$$

$$\begin{aligned} &\text{If } x(t) \text{ is } A_3 \text{ and } v(t) \text{ is } B_3 \text{ and } i(t) \text{ is } C_3 \\ &\text{then } f_3(t) = p_3x(t) + q_3v(t) + r_3i(t) + s_3 \end{aligned} \quad (4)$$

where $x(t)$, $v(t)$, and $i(t)$ are input language variables; A_j , B_j , and C_j are fuzzy sets; $f_1(t)$, $f_2(t)$ and $f_3(t)$ are output language variables; p_j , q_j , r_j , and s_j are the output parameters of the fuzzy system, then Fig. 2 would represent the *ANFIS* structure for the first-order fuzzy system. The W_j and Wn_j represent the degree of fitness and the normalized fitness of the fuzzy rules, respectively.

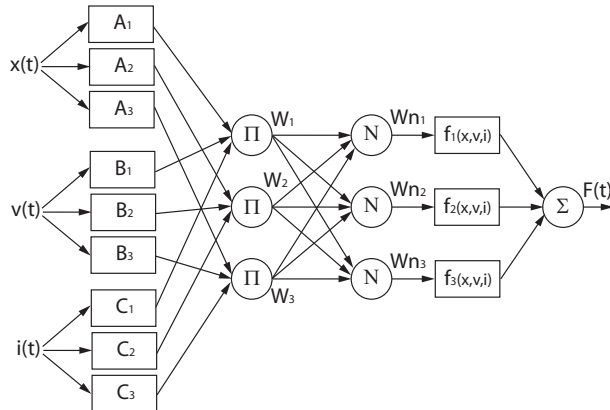


Fig. 2 *ANFIS* structure of a first-order fuzzy model with three inputs and one output. For simplicity, only three fuzzy rules, out of the 27 possible combinations, are considered.

3 Experimental System

A Delphi MagneRideTM *MR* damper was used to perform a total of 29 tests [10]. An MTSTMGT controller testing system was used to control the position of the damper. A FlextestTM data acquisition system commanded the controller and recorded the position and force of the *MR* damper, as well as the electric current on the coil. A sampling frequency of 512 hertz was

used. The bandwidth of displacement was 0.5 - 14.5 Hz, which lies within normal automotive applications. The experimental setup is shown in Fig. 3.

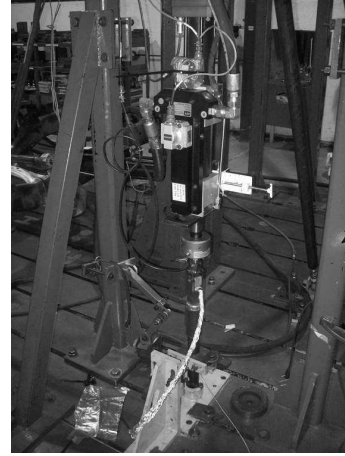


Fig. 3 Experimental setup.

Eight of the 29 tests were chosen for this study. In the experiments, the electric current, $i(t)$, patterns were Stepped Increments (*SC*), Increased Clock Period Signal (*ICPS*), Pseudo Random Binary Signal (*PRBS*), and Amplitude *PRBS* (*APRBS*). Road Profile (*RP*) signals were used as the displacement, $x(t)$, input pattern. The *RP* signals reach a maximum amplitude of 0.5 in and emulate the dynamics of a damper used in automotive applications. Various replicates of the experiments were performed and used as validation data. The specific patterns of the eight experiments are shown in Tab. 2. Fig. 4 shows the patterns used for experiments three and four. For experiments six to eight, stepped increments of 0, 0.4, 0.8, 1.2, 1.6, 2.1, and 2.5 A, each with a duration of 30 seconds, were utilized. The seven replicates for the last three experiments correspond to the seven constant current stepped increments.

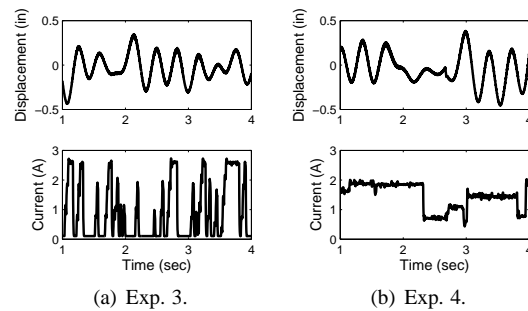


Fig. 4 Displacement and electric current patterns for experiments 3 and 4. Both experiments were done using an *RP* displacement pattern and *PRBS* and *APRBS* electric current signals, respectively.

4 ANFIS Model

An *ANFIS* structure was proposed for modelling the *MR* damper. Displacement, velocity, and electric cur-

Tab. 2 Experiments

Exp.	Displacement Pattern	E. Current Pattern	Replicates
1	Smooth Highway RP	ICPS	11
2	Smooth Highway RP	APRBS	11
3	Smooth Highway RP	PRBS	11
4	Long Duration RP	APRBS	3
5	Long Duration RP	ICPS	4
6	RP	SC	7
7	RP	SC	7
8	RP	SC	7

rent were used as inputs, and the damper force was the output. The model resembles the one in Fig. 2, but contains 27 fuzzy rules for all possible combinations of inputs. Three Gaussian membership functions were utilized to fuzzify each input. The outputs of the system were selected as linear functions.

One *ANFIS* model was trained using the first replicate of each set of experimental data after being normalized. The training was done until the error decreased by less than a threshold. Then, the eight trained *ANFIS* models were validated using the experimental data. The *ANFIS* models were named after the experiment with which they were trained. Tab. 3 presents the average *ESR* obtained by the models when validated with the experimental data sets.

The box plots on Fig. 5 and 6 show the *ESR* obtained by models 1 and 7. The figures present a box and whisker plot with one box for each experiment. The boxes have lines at the lower quartile, median, and upper quartile values. The whiskers are lines extending from each end of the boxes to show the extent of the rest of the data. Outliers are data with values beyond the ends of the whiskers.

Tab. 3 Average *ESR* by *ANFIS* model and experiment.

Exp	Model							
	1	2	3	4	5	6	7	8
1	0.07	0.15	0.12	0.22	0.21	0.10	0.10	0.10
2	0.09	0.06	0.11	0.10	0.10	0.08	0.08	0.08
3	0.33	0.36	0.08	0.35	0.19	0.10	0.10	0.10
4	0.07	0.07	0.09	0.07	0.06	0.07	0.07	0.07
5	0.09	0.12	0.10	0.12	0.08	0.08	0.08	0.08
6	0.22	0.90	0.25	0.78	0.45	0.06	0.08	0.09
7	0.53	1.69	0.68	2.04	1.99	0.09	0.07	0.12
8	1.02	8.96	1.07	1.47	0.62	0.11	0.07	0.05

5 Non-Linear Fuzzy Model

A TSK non-linear fuzzy model [11] was proposed for the *MR* damper. Using the electric current as input for the model, fuzzy rules were proposed as specified in Eq 5.

$$\text{If } i(t) \text{ is } A_j \text{ then } f(t)_j = g_j(x(t), v(t)) \quad (5)$$

Notice that each output function $f(t)_j$ depends on the displacement and the velocity of the *MR* damper. A_j

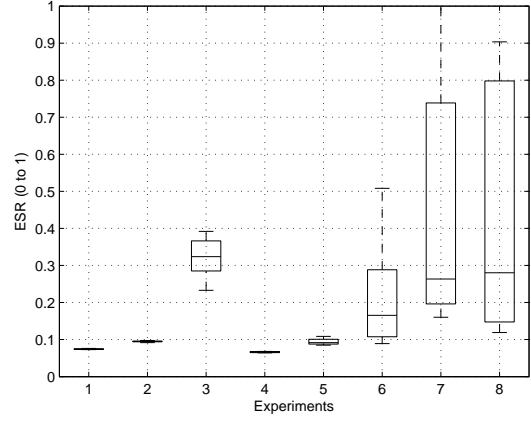


Fig. 5 *ESR* by experiment for *ANFIS* model 1. The error of the model greatly increased when validated with experiments 3 and 6 to 8.

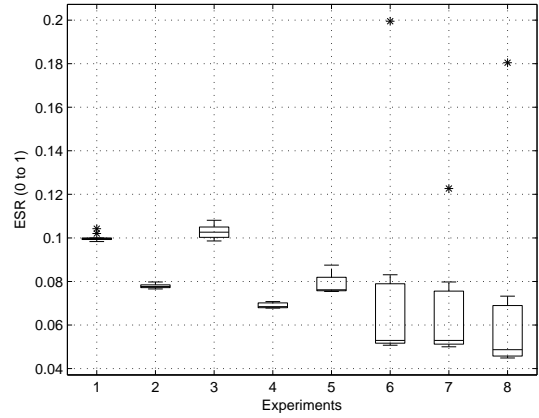


Fig. 6 *ESR* by experiment for *ANFIS* model 7. The error of the model remained below 0.1 for all experiments. The outliers on the validation with experiments 6 to 8 correspond to zero input electric current.

are fuzzy sets of i . The output functions were selected to be of the form of the semi-phenomenological model for the *MR* damper in [9]. The model is shown in Eq. 6,

$$f(t)_j = c_{1j} \tanh(c_{2j} (v(t) + c_{3j} x(t))) + c_{4j} (v(t) + c_{3j} x(t)) \quad (6)$$

where the coefficients c_{1j} , c_{2j} , c_{3j} , and c_{4j} are to be determined from experimental data.

The overall output force of the damper is computed as specified by Eq. 7,

$$F(t) = \frac{\sum_{j=1}^7 W_j(i(t)) f(t)_j}{\sum_{j=1}^7 W_j(i(t))} \quad (7)$$

where W_j represents the membership degree of $i(t)$ on each of the membership functions. As Eq. 6 only depends on the displacement and velocity of the damper, the coefficients were identified using experiments six to eight. The fitting algorithm selected was non-linear least squares and yielded one non-linear equation for each of the seven electric current stepped increments on the experiments. In this way, one non-linear fuzzy model was obtained from experiment 6, one from experiment 7, and one from experiment 8. The fuzzy models were labeled according to the experiments with which they were trained. Fig. 7 depicts the proposed fuzzy structure.

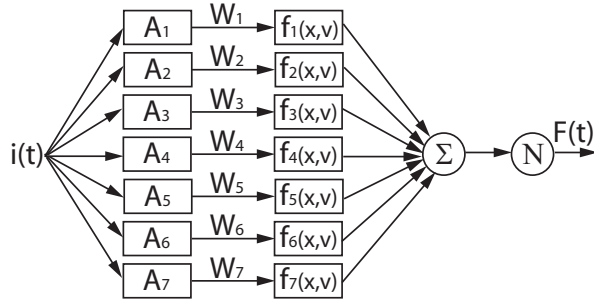


Fig. 7 Non-linear fuzzy model structure with one input and one output. Seven possible input membership functions are considered.

The input membership functions for each model were defined as seven Gaussian functions with a variance equal to 0.2 and means of 0, 0.4, 0.8, 1.2, 1.6, 2.1, and 2.5 A, respectively. Additionally, seven output functions were selected in the form of Eq. 6 with coefficients previously identified.

The proposed models were validated using the eight sets of experimental data. The box plots on Figs. 8 - 10 present the resulting ESR by experiment.

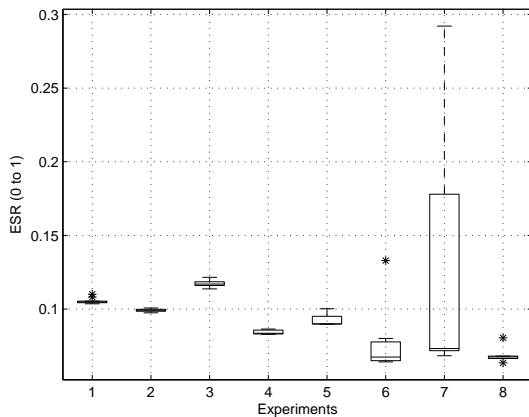


Fig. 8 ESR by experiment for fuzzy model 6. The error of the model remained below 0.12 for most of the experiments. The variance greatly increased for experiment 7.

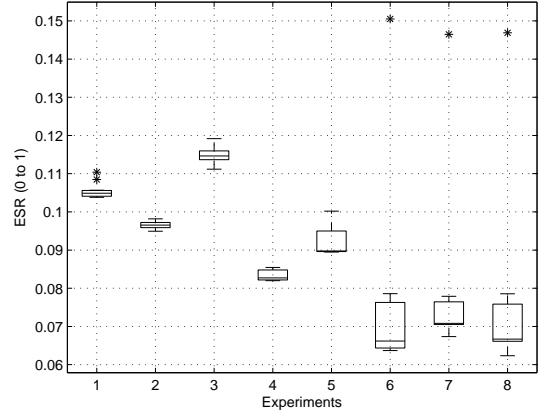


Fig. 9 ESR by experiment for fuzzy model 7. The error of the model constantly remained below 0.12. The outliers on the validation with experiments 6 to 8 correspond to zero input electric current.

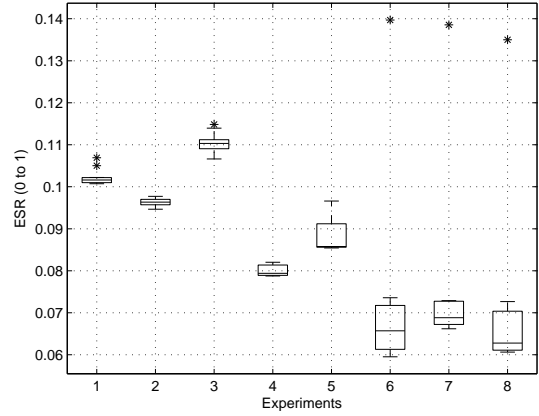


Fig. 10 ESR by experiment for fuzzy model 8. The error of the model constantly remained below 0.11. The outliers on the validation with experiments 6 to 8 correspond to zero input electric current.

6 Discussion

The *ANFIS* structure obtained ESR values of approximately 0.1 when trained using experimental data with constant electric current stepped increments. Nonetheless, the ESR variance increased considerably when validated with constant electric current experiments (Fig. 6). The outliers on the plot correspond to experiments with zero electric current. On the other hand, *ANFIS* structures trained using experimental data with varying electric current could not predict the output force of the damper when validated with constant electric current inputs.

Surprisingly low ESR values were constantly obtained by the proposed non-linear fuzzy structure. Excellent results were seen with fuzzy model 6, except when validated with experiment 7. With fuzzy models 7 and 8

the outliers observed on the resulting *ESR* validation box plots correspond to experiments with zero electric current. Additionally, for both proposed structures, the *ESR* variance increased when validated with constant electric current experiments.

Tab. 4 presents the average *ESR* obtained by both the *ANFIS* and non-linear fuzzy structures. Notice that, if classified by the average error, *ANFIS* models 6 to 8 and the three fuzzy models are the best options for modelling the *MR* damper.

Tab. 4 Average *ESR* by model.

Model	Average <i>ESR</i>
<i>ANFIS</i> 1	0.30
<i>ANFIS</i> 2	1.44
<i>ANFIS</i> 3	0.30
<i>ANFIS</i> 4	0.62
<i>ANFIS</i> 5	0.45
<i>ANFIS</i> 6	0.09
<i>ANFIS</i> 7	0.08
<i>ANFIS</i> 8	0.09
Fuzzy 6	0.10
Fuzzy 7	0.09
Fuzzy 8	0.09

When comparing the complexity of the structures, the proposed non-linear fuzzy model is clearly less complex than the *ANFIS* one. The first is composed of 27 rules, whereas the latter contains only seven. Nonetheless, the advantage of the *ANFIS* structure lies in its use of simple linear output functions instead of non-linear ones.

The present study confirmed the work done in [5], [6], and [7] for model fitting and extended it to experimental data. As mentioned in [6], it was proved that an *ANFIS* structure can successfully capture the dynamics of an *MR* damper and can be a useful tool for control. In addition, the results obtained by the proposed non-linear fuzzy structure extended the capabilities of the model in [9] to varying electric current scenarios.

Further work may address the use of different phenomenological models of the *MR* damper as output functions for the non-linear fuzzy structure. Additionally, the non-linear fuzzy model may benefit from the inclusion of more input membership functions.

7 Conclusion

The results obtained showed that fuzzy-based modelling can be a powerful method for describing the behavior of highly non-linear systems. As opposed to phenomenological techniques, fuzzy methods do not require a profound knowledge of the dynamics of the system. After validating the *ANFIS* structures, experiments with stepped increments of the electric current allowed for a better training of the models and obtained average *ESR* values of less than 0.1. The proposed non-linear fuzzy structure successfully combined fuzzy methods with semi-phenomenological modelling. The

three trained non-linear fuzzy models obtained average *ESR* values of less than 0.1.

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